Microsoft Research

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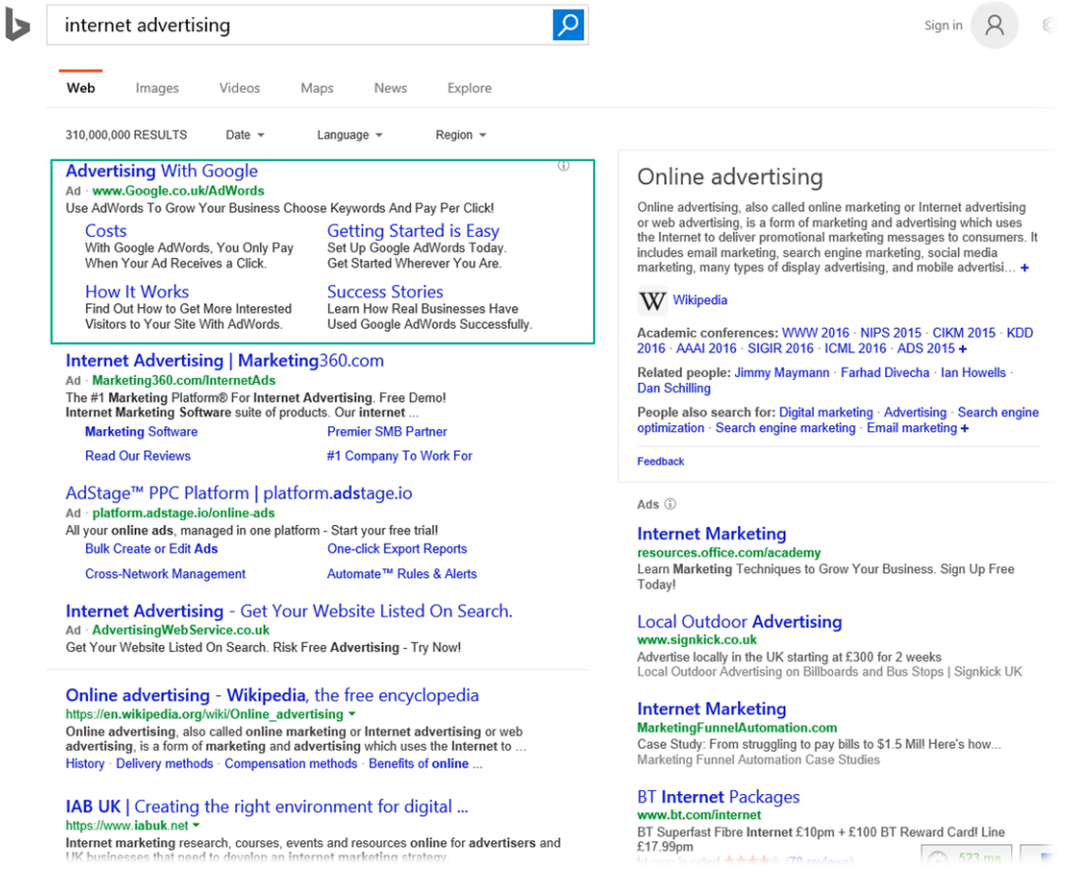
Mechanism Design for Mixed Ads

Yoram Bachrach, Sofia Ceppi, Ian Kash,
Peter Key, Mohammad Reza Khani

Mechanisms for Internet Advertising

- Search: GSP
- Display: Second Price



(GSP=Generalized Second Price)

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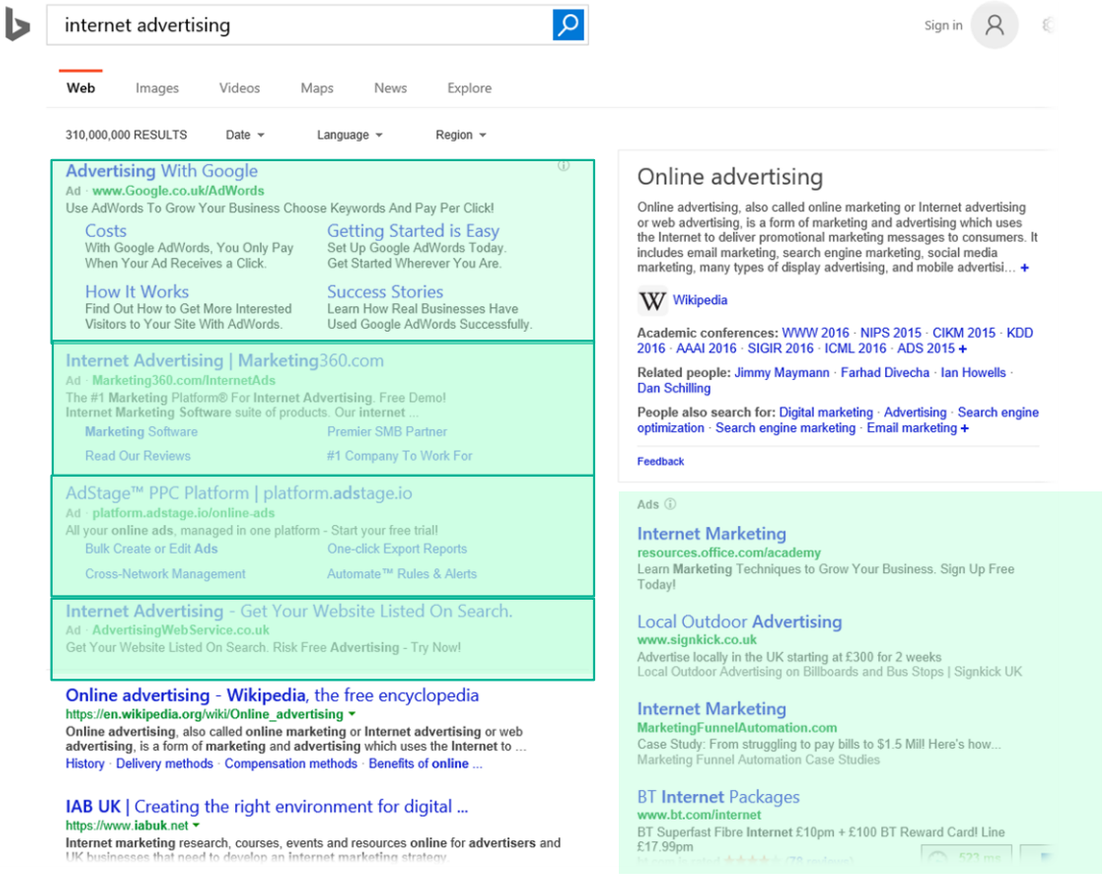


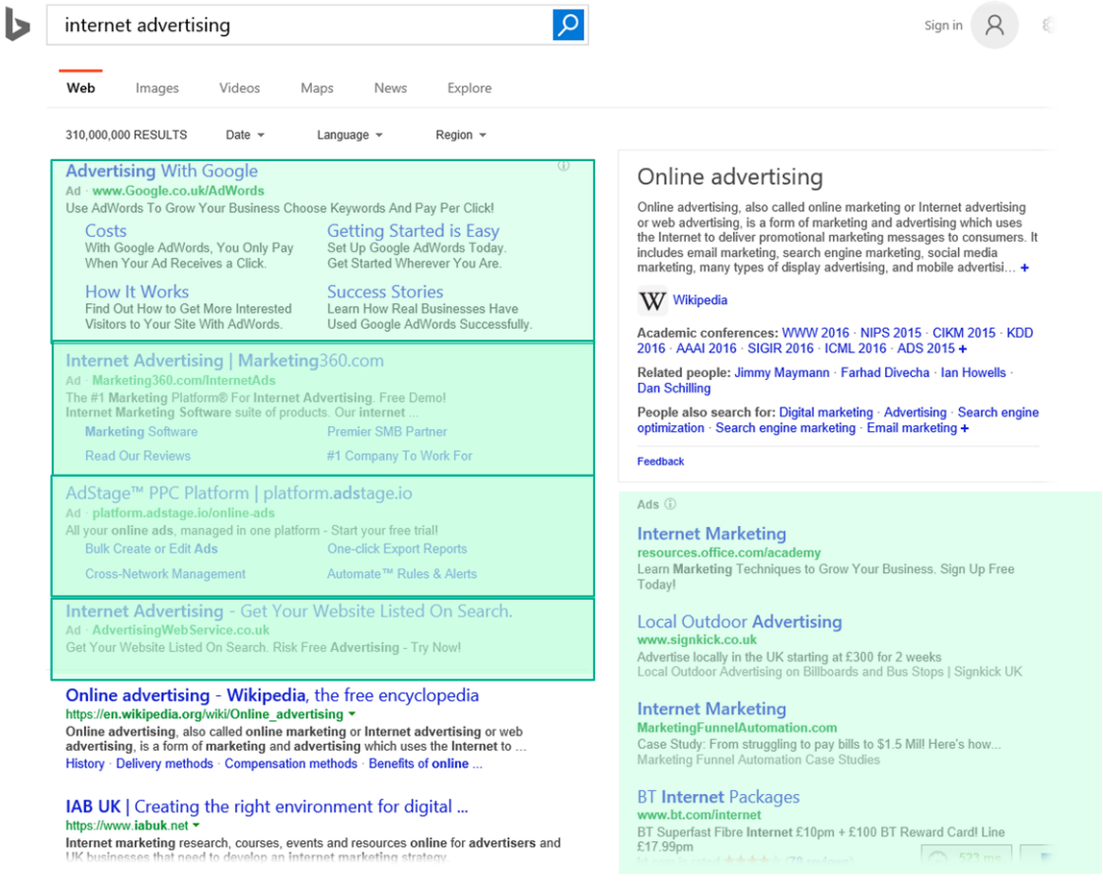
(GSP=Generalized Second Price)

Mechanisms for Internet Advertising

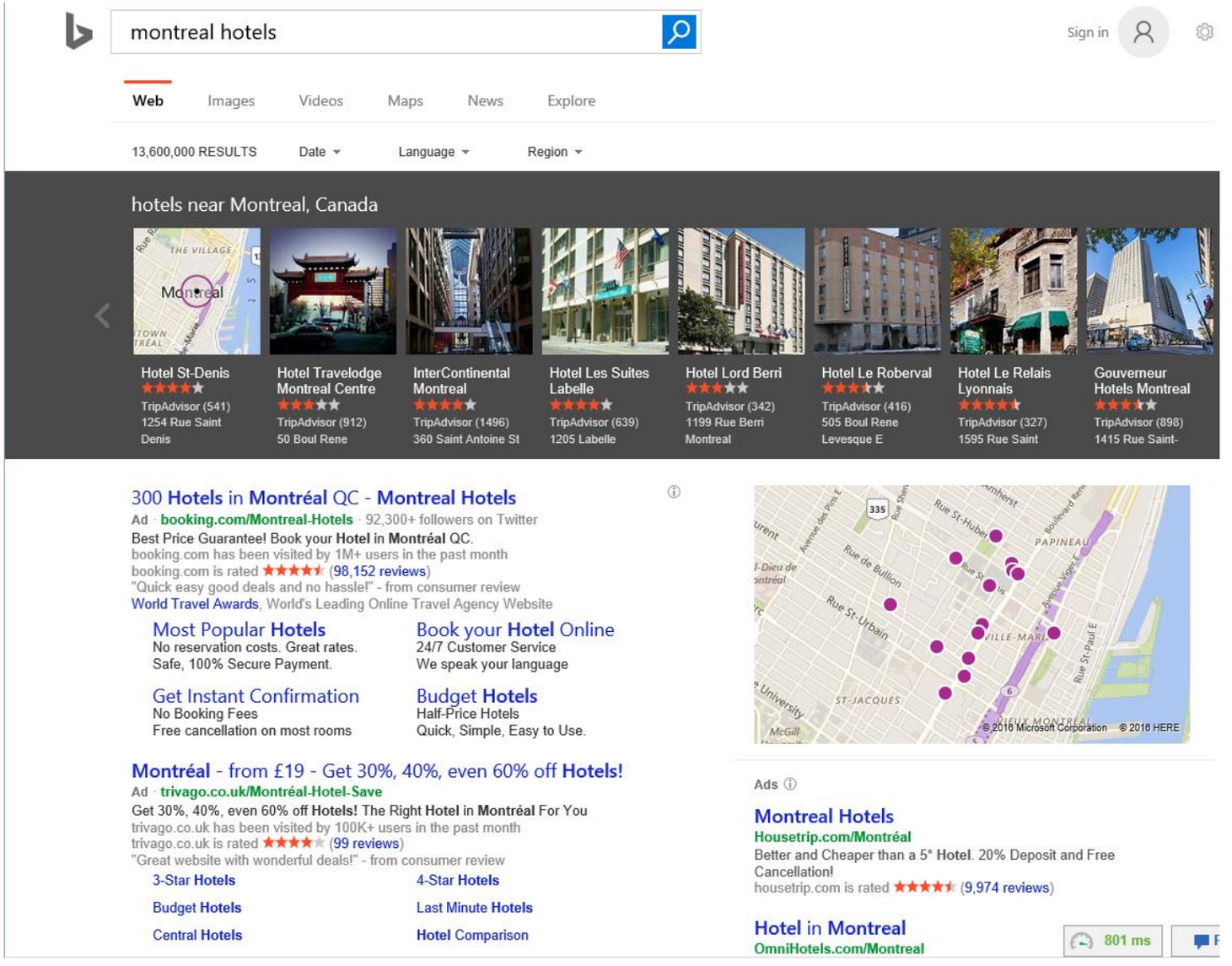
- Search: GSP
- Display: Second Price

- Facebook: VCG
- Google: VCG for Contextual Ads [Varian and Harris '14]

(VCG=Vickrey-Clarke-Groves)
(GSP=Generalized Second Price)



Problems with GSP



Problems with GSP

- Not well-defined across templates [Bachrach et al. 2014]
- Efficiency loss in broad match
- Harder to optimize

Is the transition GSP → VCG possible?

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- advertisers may not update their bid quickly
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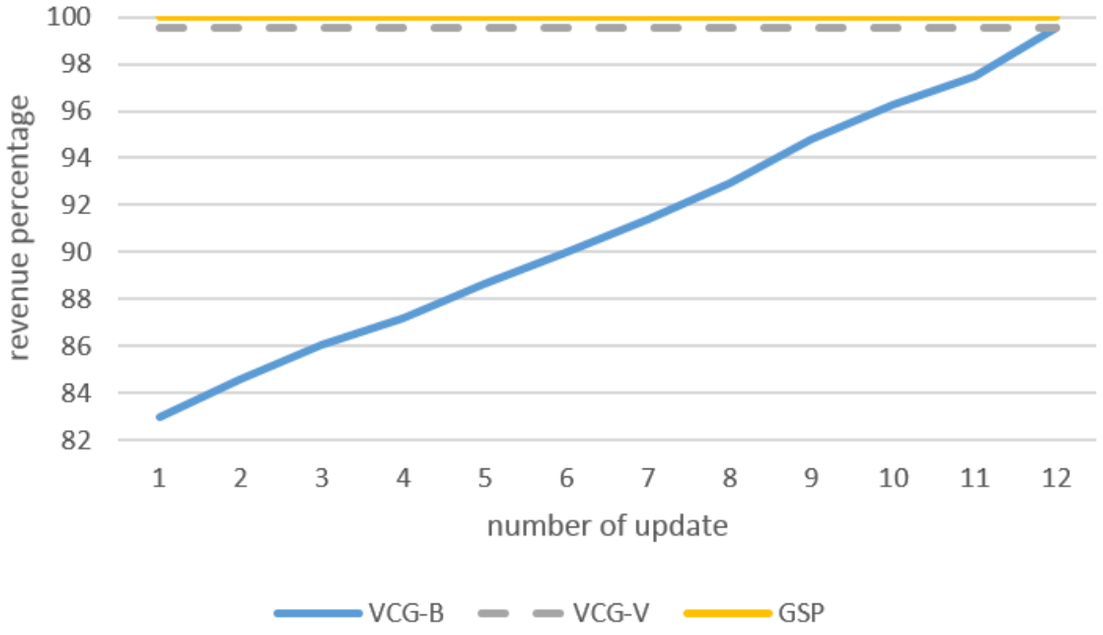

Plot Spoiler

- Hybrid auction is truthful for “truthful” bidders
- Bidders want to become “truthful”: optimal for them
- No one pays more than her bid
- Equivalent to current GSP if all bidders are GSP

Ideal Transitional Mechanism

- behaves as a GSP auction in the absence of VCG ads
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Maintain a classification of ads and treat them differently

The transitional mechanism has to:

- respect incentives for VCG ads
- be reasonable for GSP ads

Key Difficulty

To understand how bids of one group of ads affect allocation and payment of the other group of ads

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value = 20

bid = 10

value = ???

T="Truthful" ↔ Adaptive

Vert ↔ véridique ↔VCG

G=GSP ↔ Non-Adaptive

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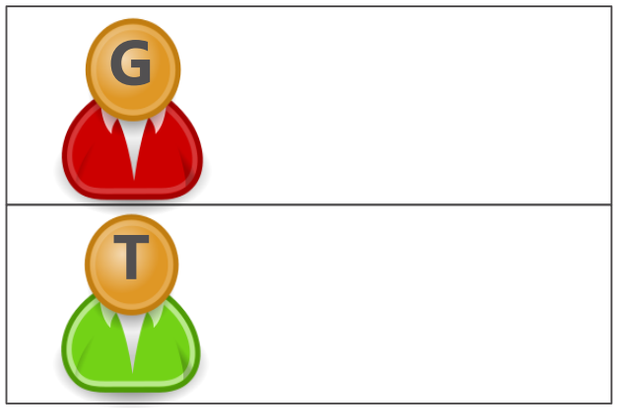

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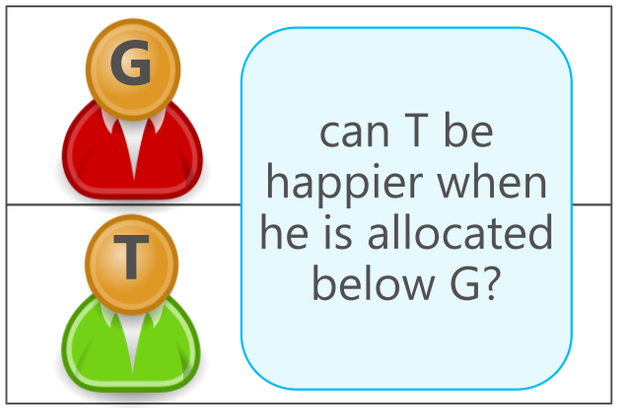

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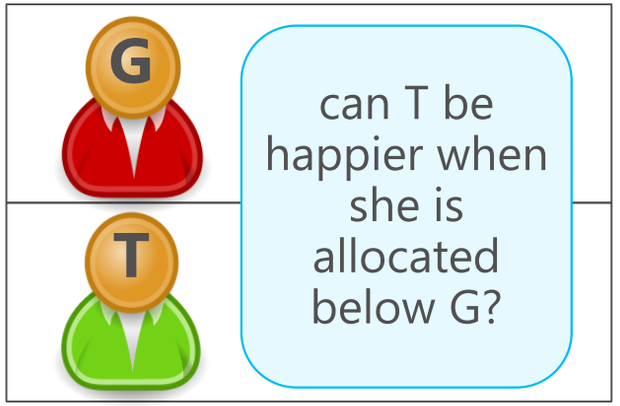

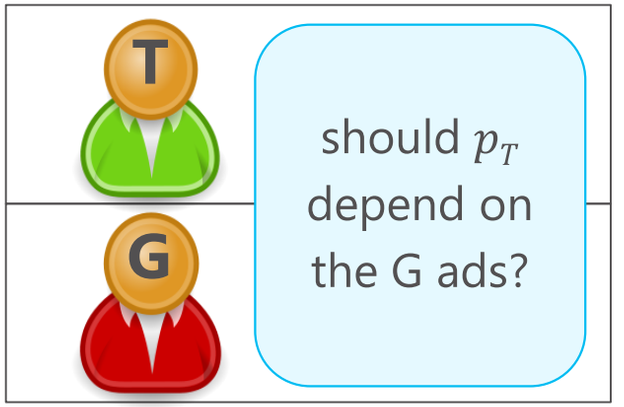

How to Overcome the Difficulty

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Usual approach to design a truthful mechanism:

1. give an allocation function
2. derive the unique payments

Our approach to design a truthful mechanism:

1. give a payment function
2. derive the allocation

Notation for slot ads

$[n] = \{1, \dots, n\}$ ads

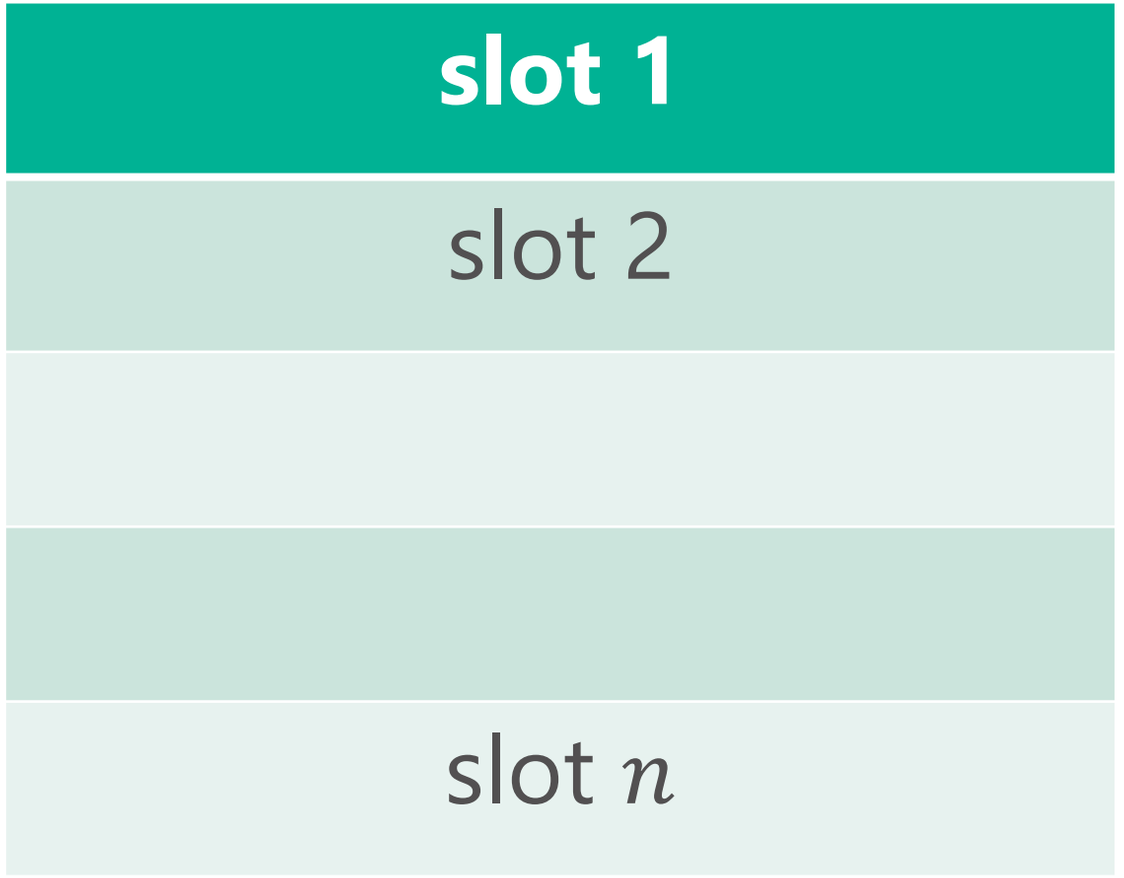
n slots (ad positions)

ad $type$:

$(label, bid) \in \mathcal{T} = \{T, G\} \times \mathbb{R}^+$

f_i is CTR of slot i

$0 < f_n < \cdots < f_1 \leq 1$



Payment Framework

Requirement:

- Valid Ads Ordering

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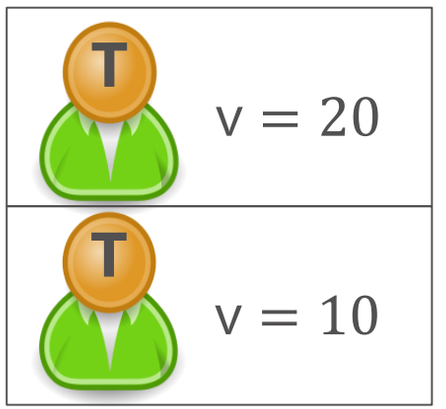

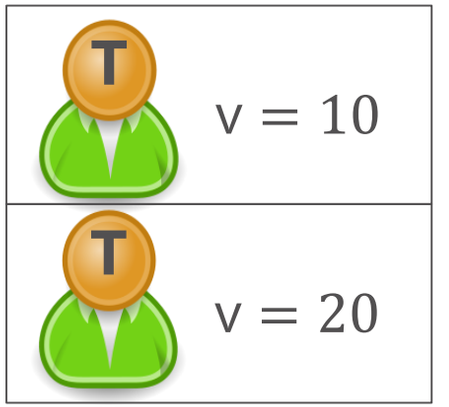

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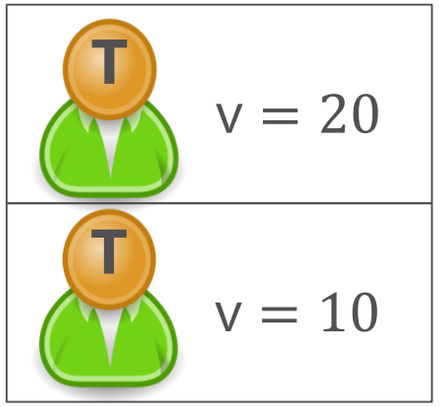

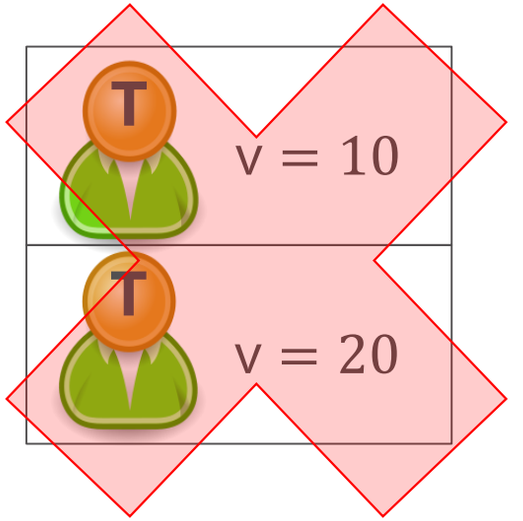

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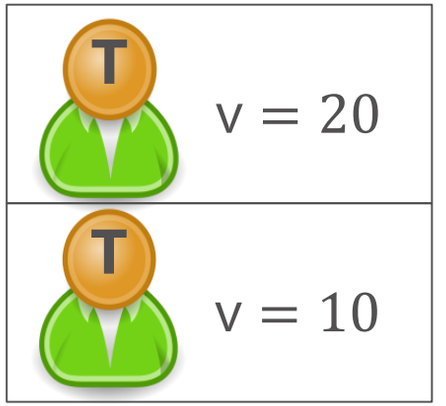
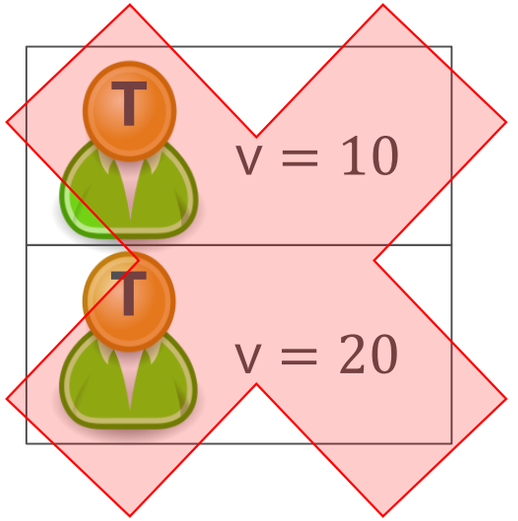




Properties:

- Top Interference Free
- Exact Marginal Increment
- Minimum Marginal Increment

Given a payment rule that satisfies these properties, there exists an allocation rule such that the resulting mechanism is truthful for truthful ads.

Payment Notation for slot ads

$[n] = \{1, \ldots, n\}$ ads , n slots

Assignment

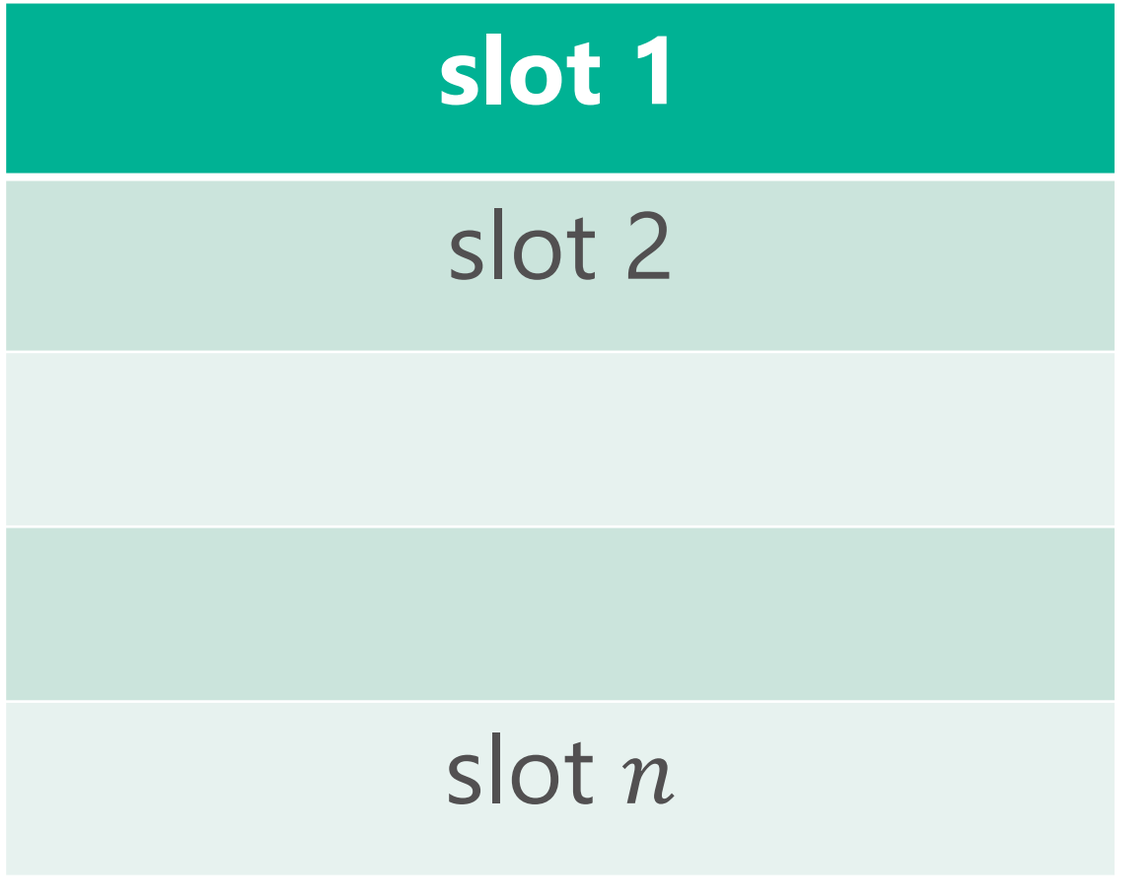
$$\Pi = \{\pi_1, \ldots, \pi_n\}$$

$\mathbf{ad}\ \pi_i$ assigned slot i

Payment

$$\mathcal{P} = \{p^{(i)}\}_{i \in [n]}$$

$$p^{(i)} : \overbrace{\mathcal{T} \times .. \times \mathcal{T}}^{n-i} \dashrightarrow \mathbb{R}^+$$



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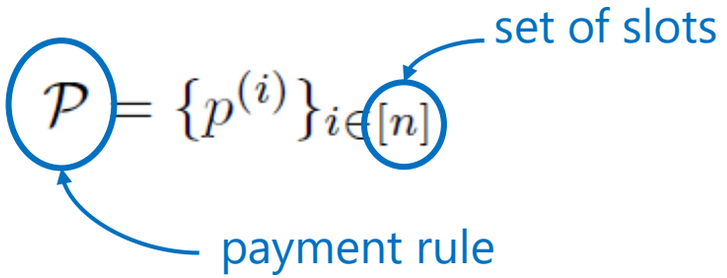

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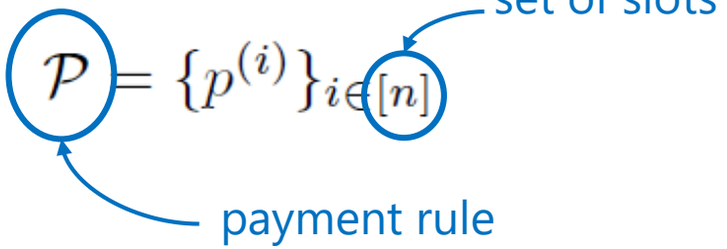

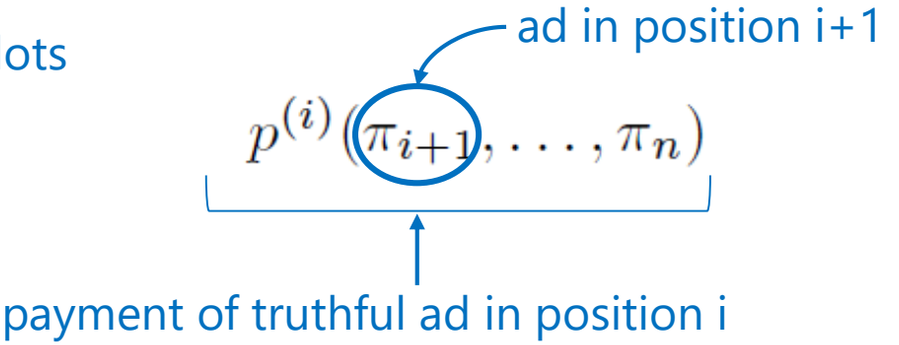

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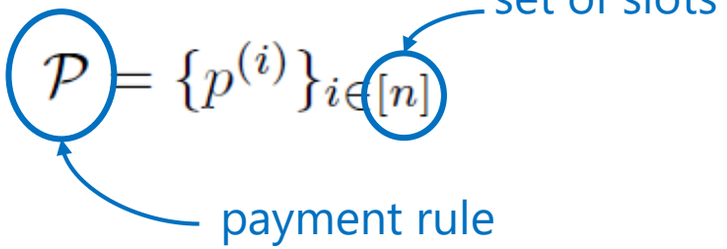

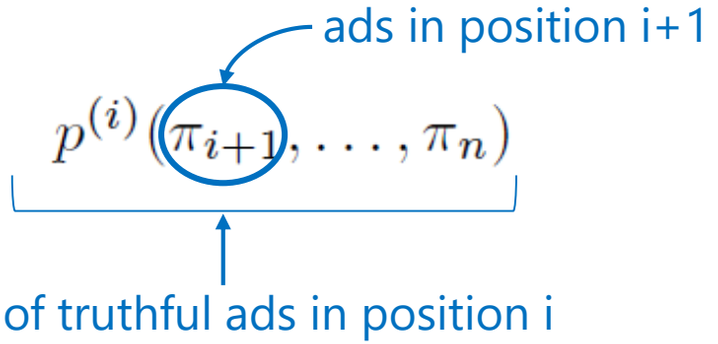

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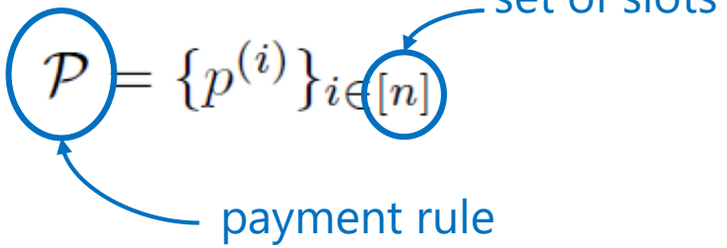
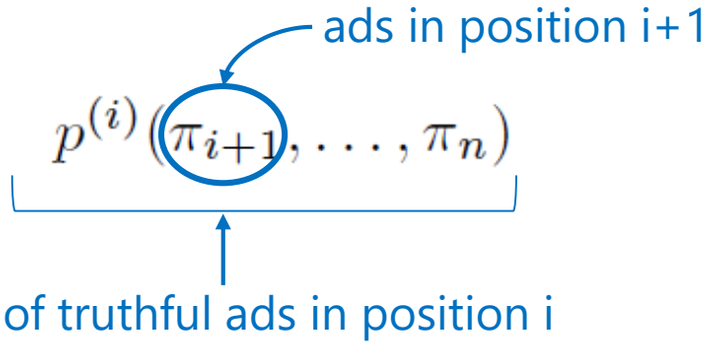
Exact Marginal Increment

For any assignment Π and position $i \in [n-1]$,
if π_{i+1} is a truthful ad then

$$\nabla^{(i,i+1)}\mathcal{P}(\Pi) = v(\pi_{i+1})$$

Top Interference Free





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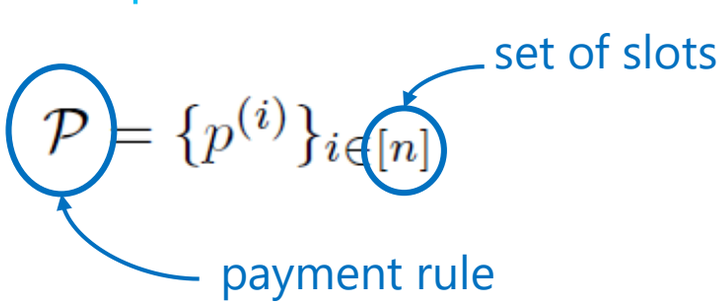
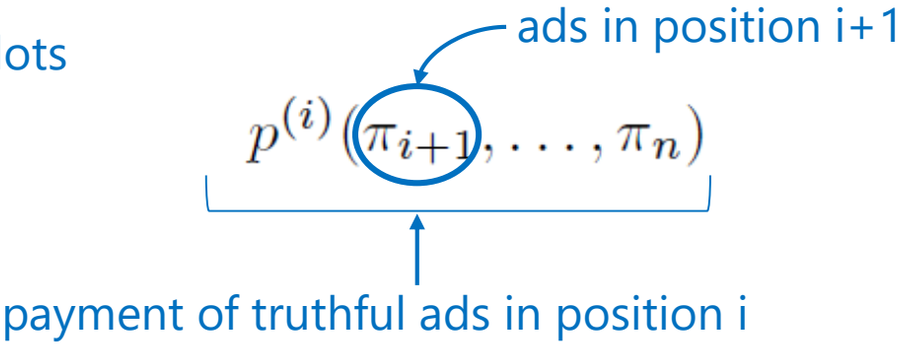
Marginal Operator

$$\nabla^{(i,j)}\mathcal{P}(\Pi) = \frac{p^{(i)}(\Pi^{(i+1)}) - p^{(j)}(\Pi^{(j+1)})}{f_i - f_j}$$

$$i < j,\ i, j \in [n]$$

Top Interference Free





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Minimum Marginal Increment

For any assignment Π and position $i \in [n]$ such that π_i is a truthful ad and slot $j \in \{1, \ldots, i-1\}$ then

$$\nabla^{(j,i)}\mathcal{P}(\Pi) \geq v(\pi_i)$$

Marginal Operator

$$\nabla^{(i,j)}\mathcal{P}(\Pi) = \frac{p^{(i)}(\Pi^{(i+1)}) - p^{(j)}(\Pi^{(j+1)})}{f_i - f_j}$$

$$i < j,\ i, j \in [n]$$

Allocation Algorithm

$v_{T1} > v_{T2}$

$b_{G1} > b_{G2}$

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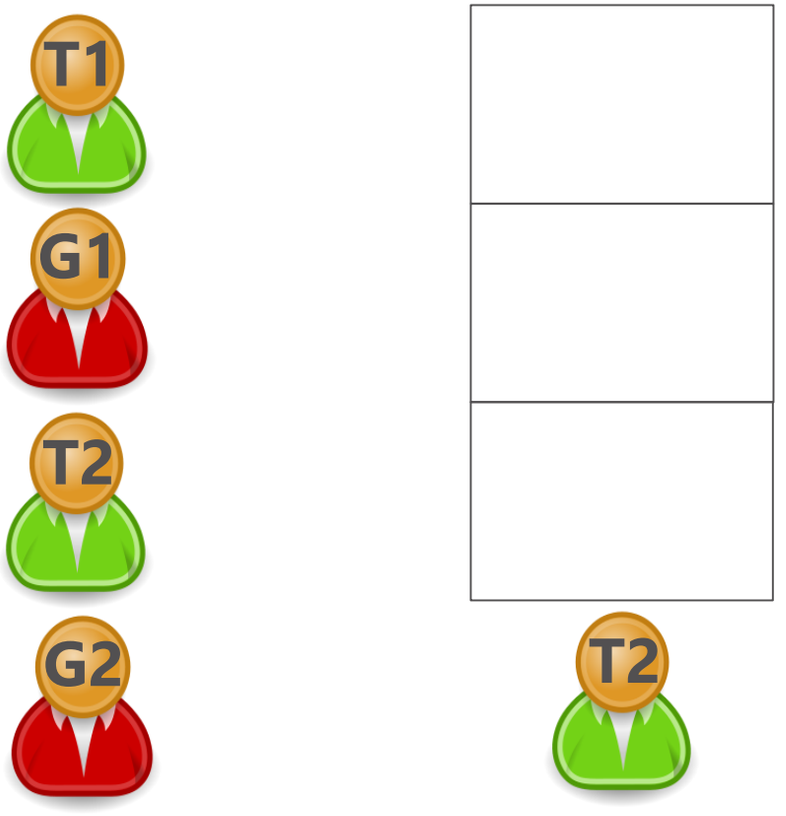

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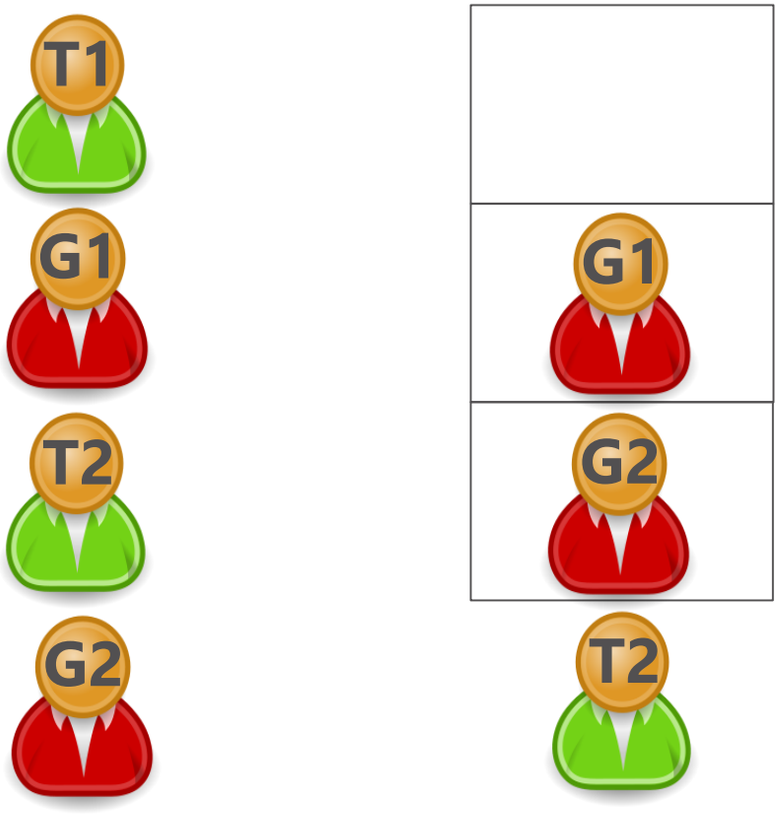

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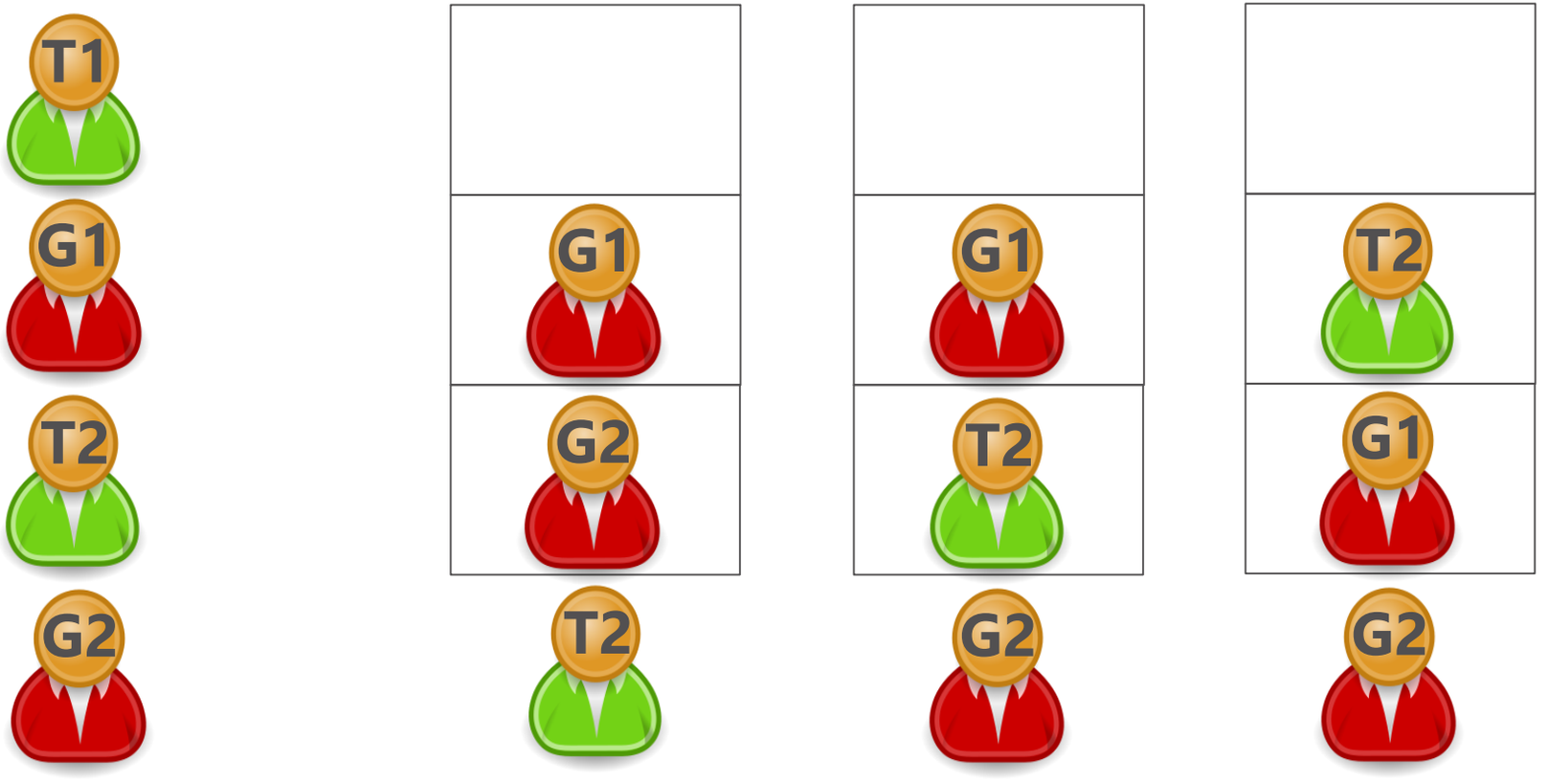

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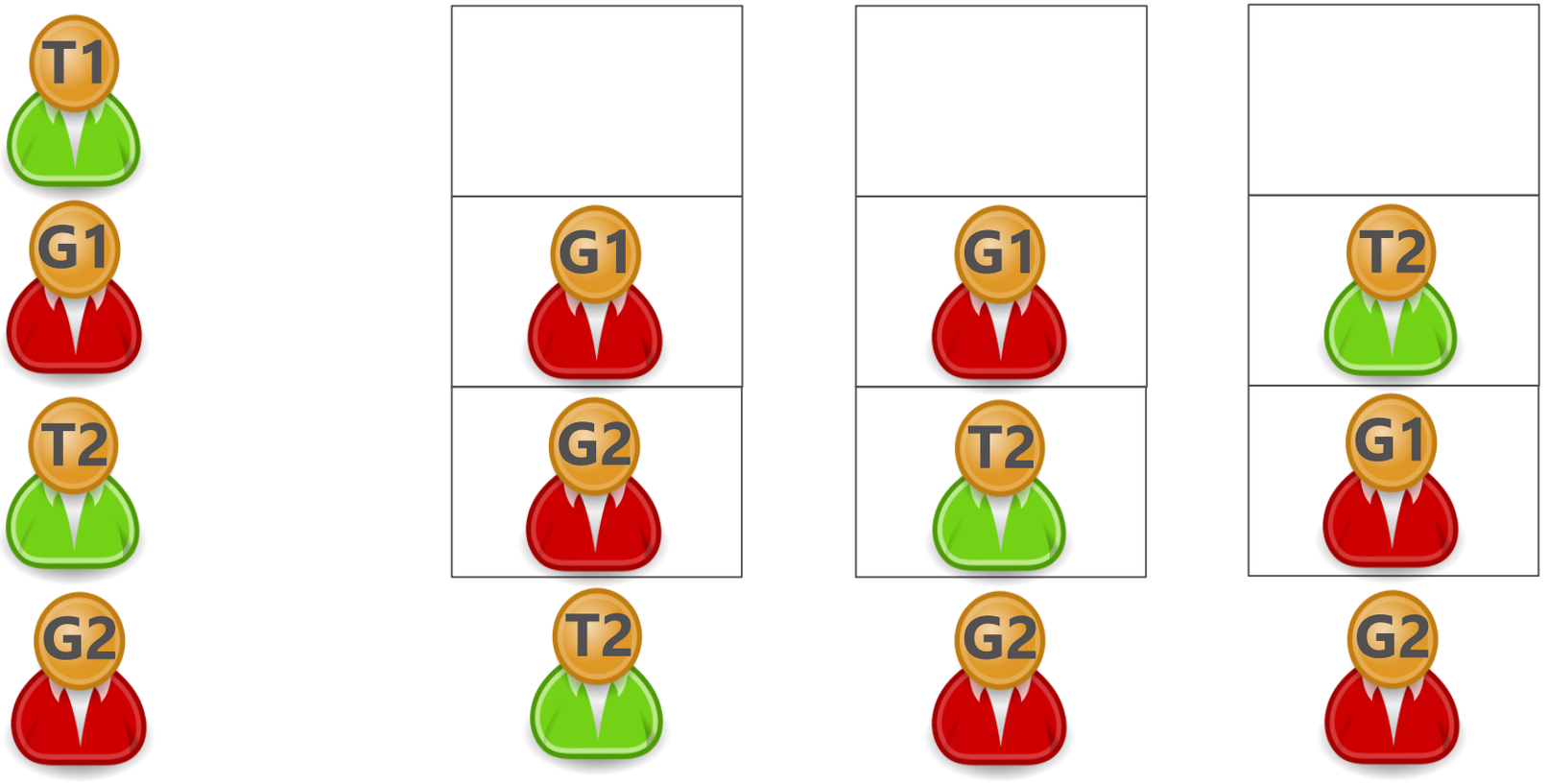

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u_{T2} u'_{T2} u''_{T2}

compute the expected utility of T2 in each of this allocation

$$u_i(b_i, b_{-i}) = \left(v_i - p^{(j)}\right) * fj$$

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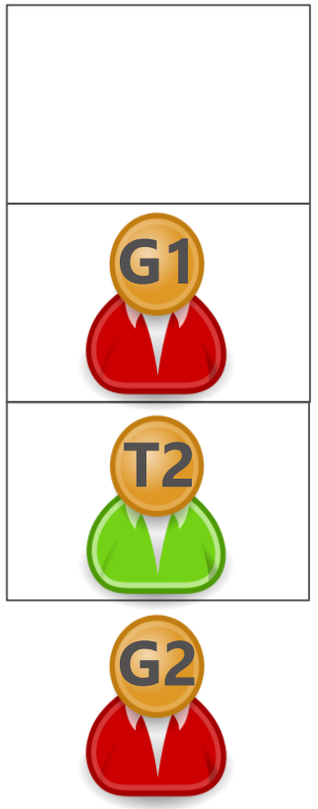
u_{T2}



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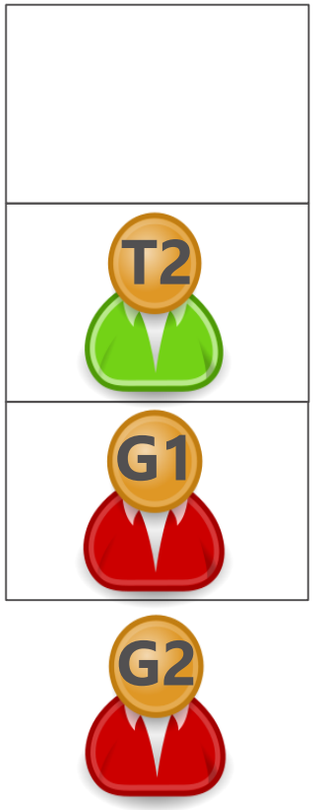

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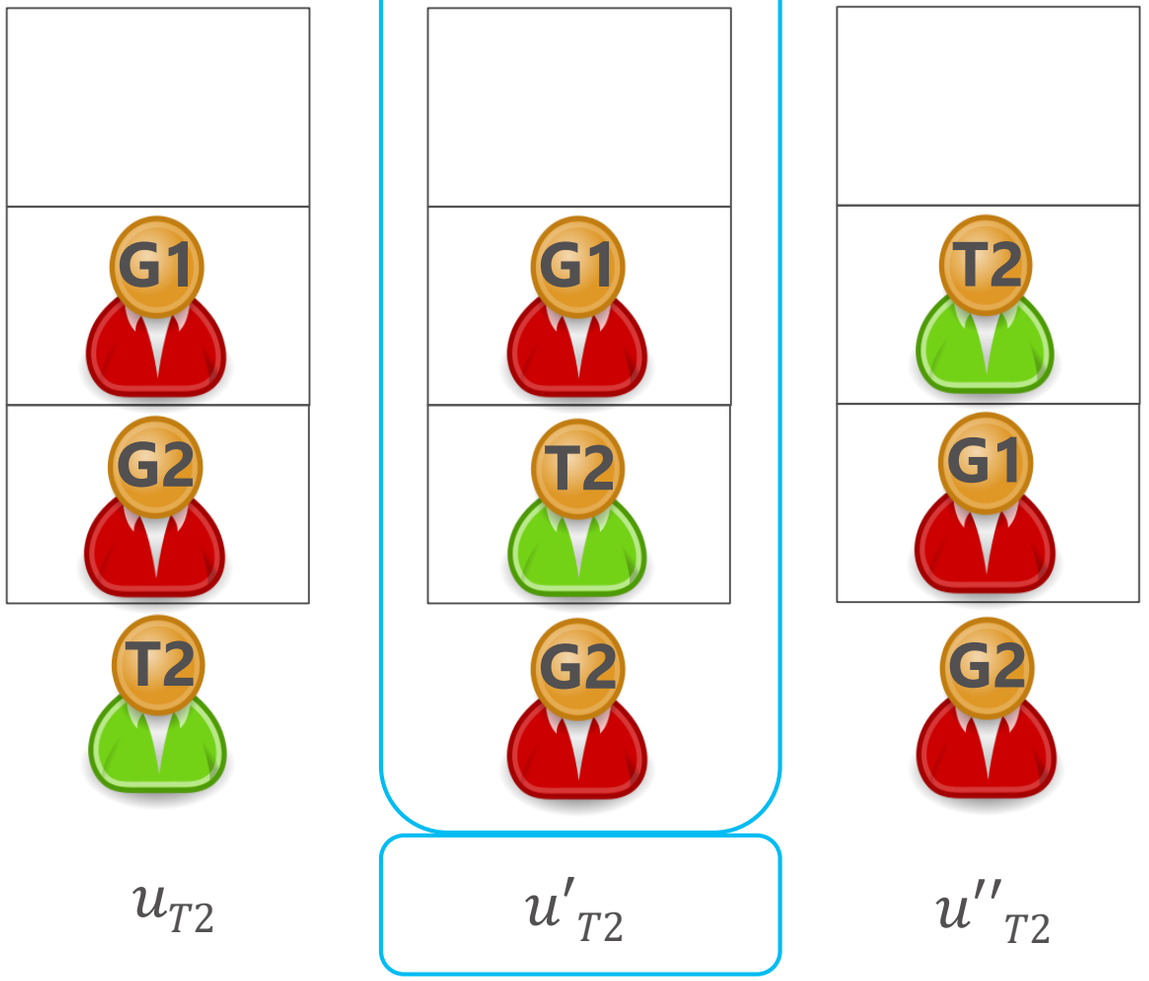

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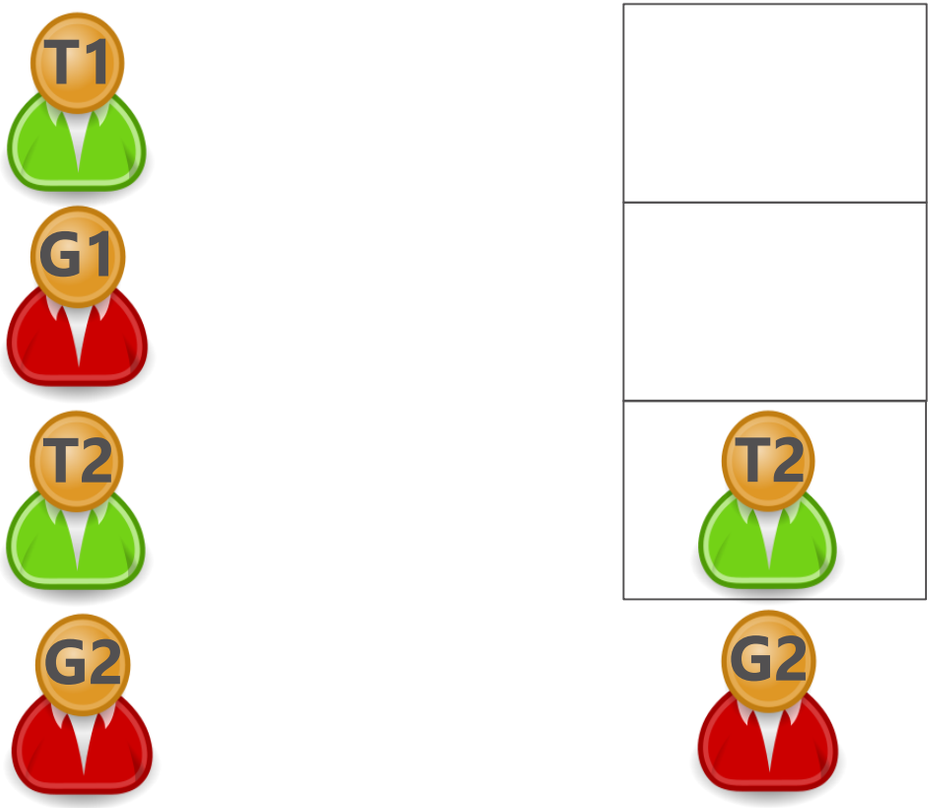

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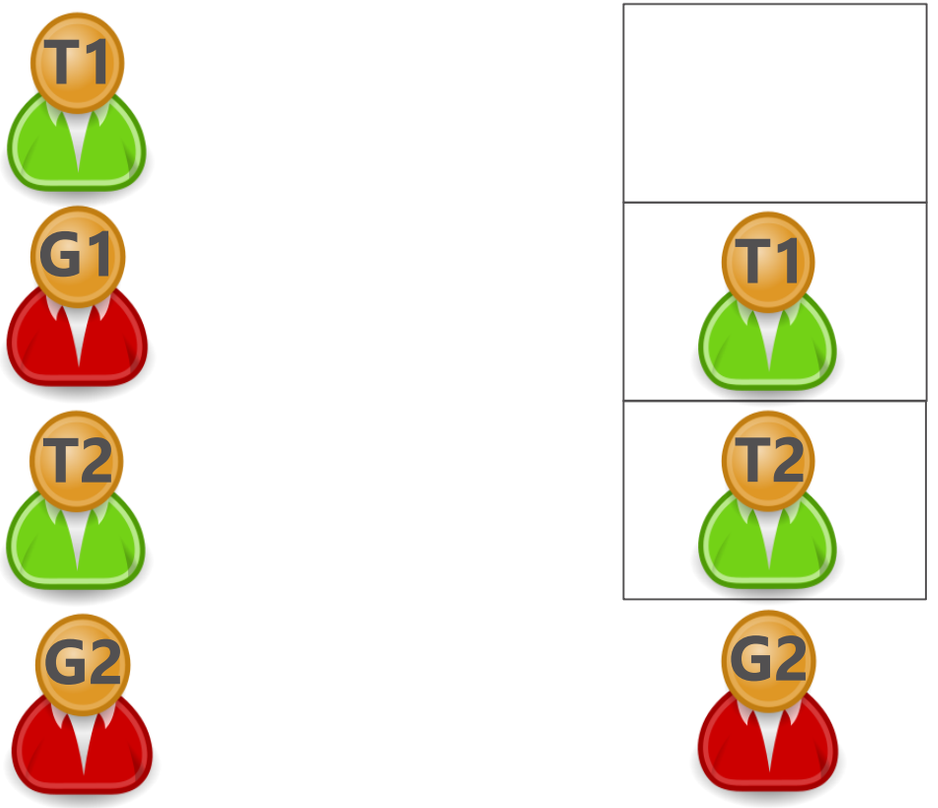

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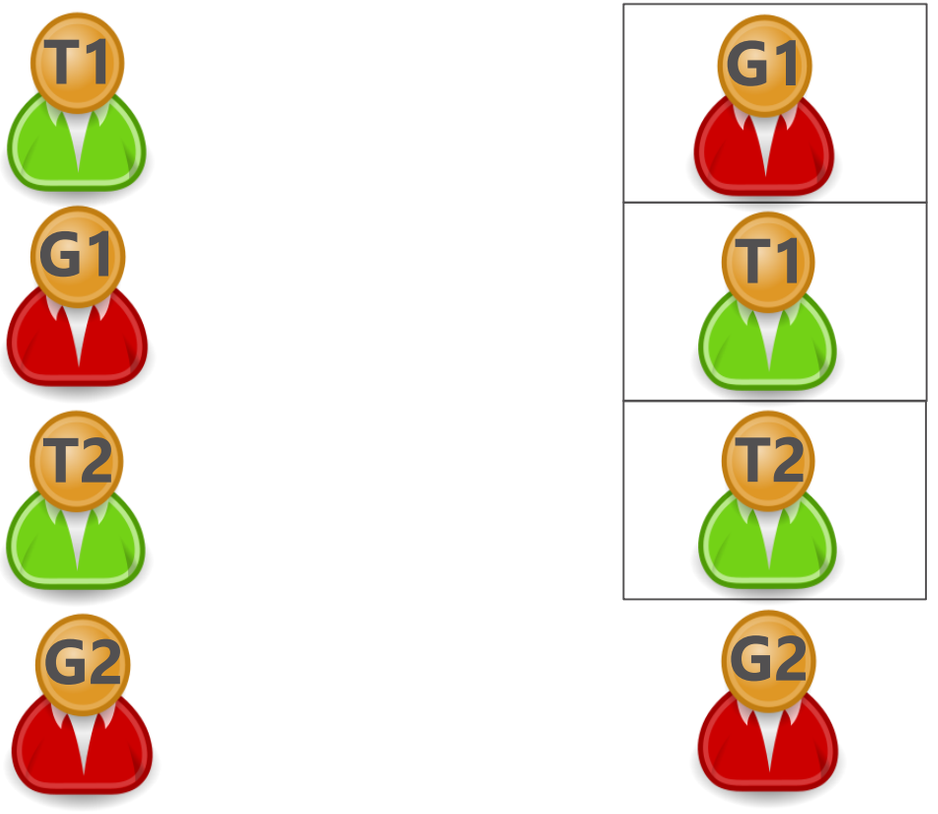

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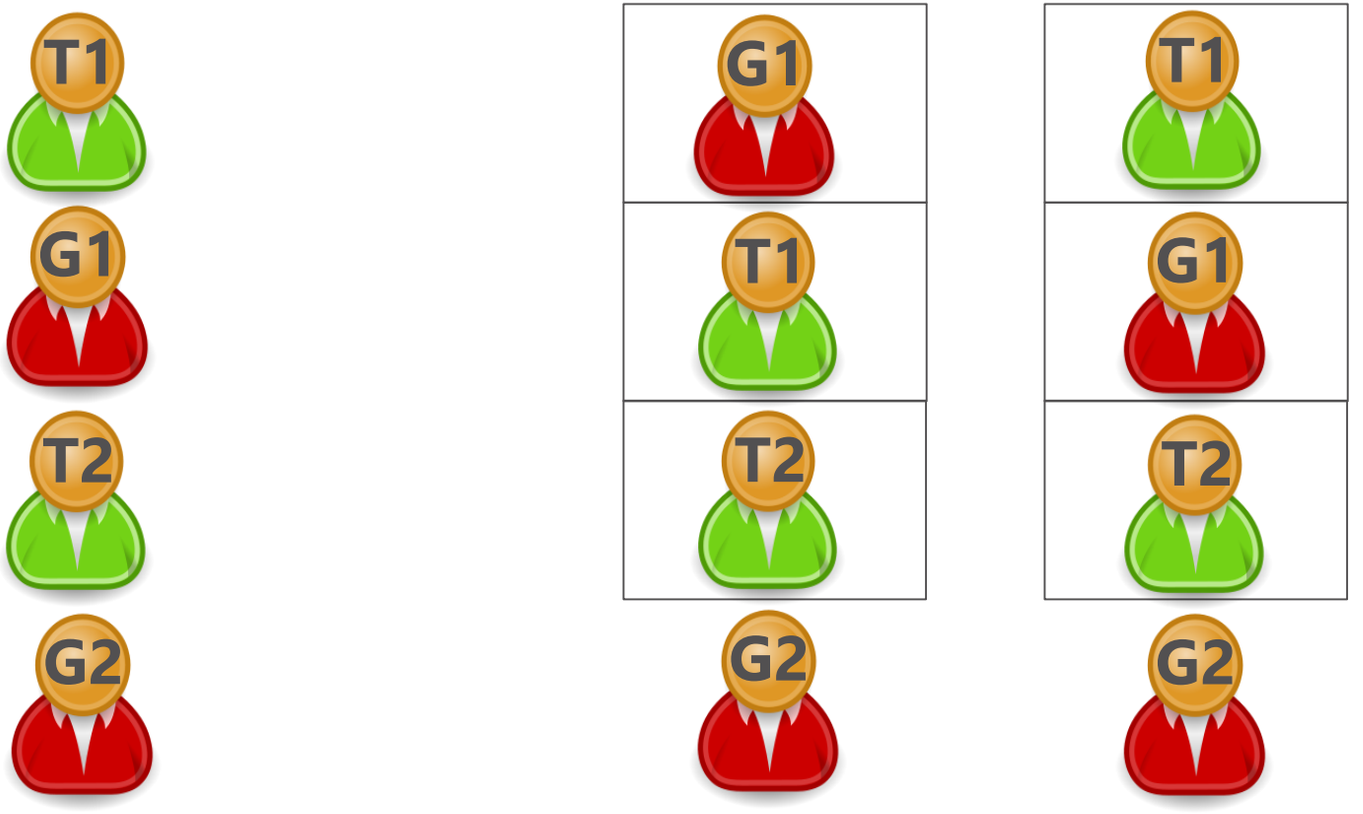

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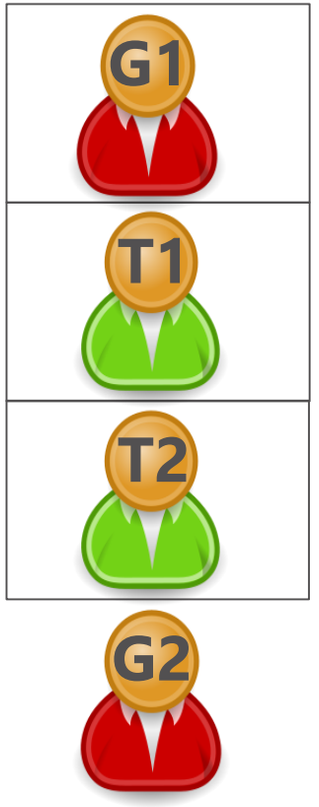

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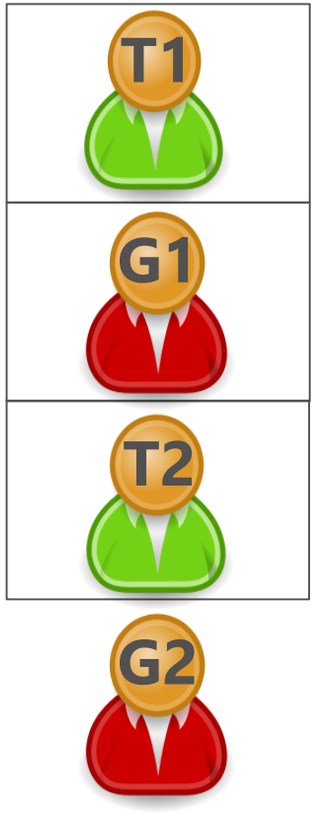

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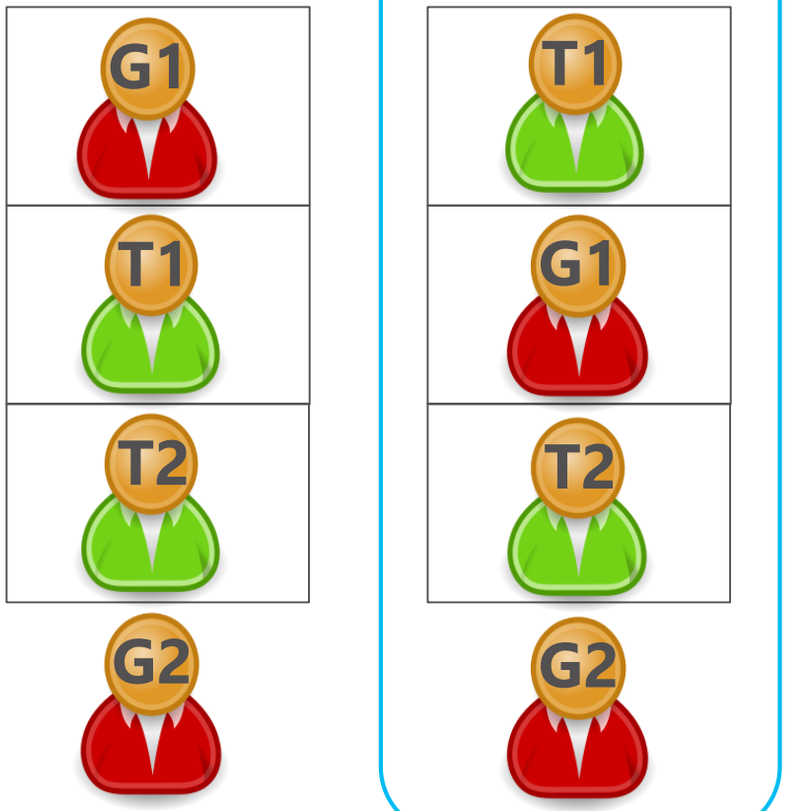
$b_{G1} > b_{G2}$



u_{T1}

u'_{T1}

$$u_i(b_i, b_{-i}) = \left(v_i - p^{(j)}\right) * fj$$

compute the expected utility of T1 in each of this allocation

ad T1 is assigned to the slot in the allocation that gives her the highest expected utility

Theorems

Given a set of payment functions *P* that satisfy EMI and MMI, the mechanism derived from applying the Allocation Algorithm to *P* is incentive compatible for truthful ads.

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A mechanism is derived from our framework if and only if it satisfies TIF and is Truthful and Anonymous.

Example of Payment Function

$$p^{(i-1)} = \max(p^{(j)} + v_j \left(f_{i-1} - f_j\right), b_k * f_{i-1})$$

j: position of the highest T ad below $i-1$
k: position of the highest G ad below i-1
v_j: value of the ad in position j
b_k: bid of the ad in position j

Simulation Results

Setup:

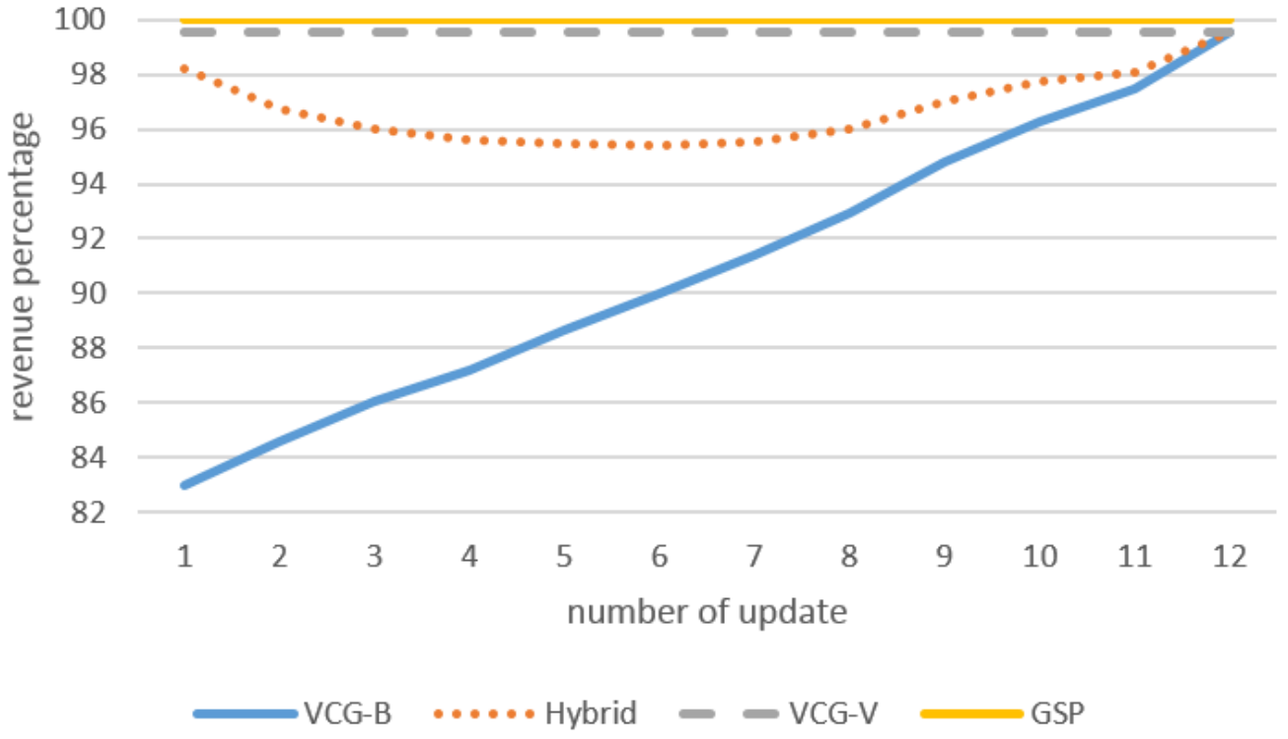
- 3984 auctions with at least 12 participants (Bing data)
- 3 top slots
- estimate of ads valuations by using the stochastic formulation proposed by Pin and Key [2011]

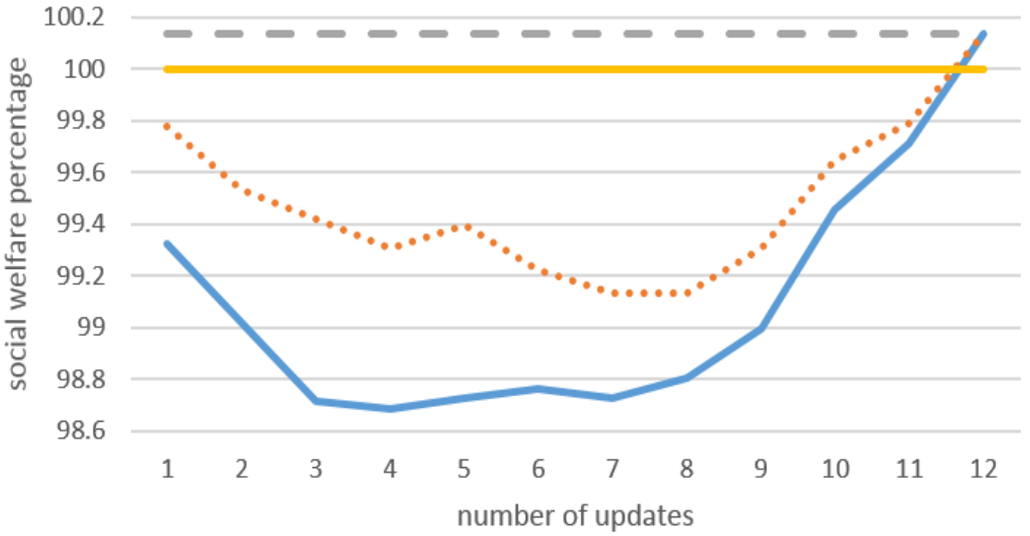
Simulation Results

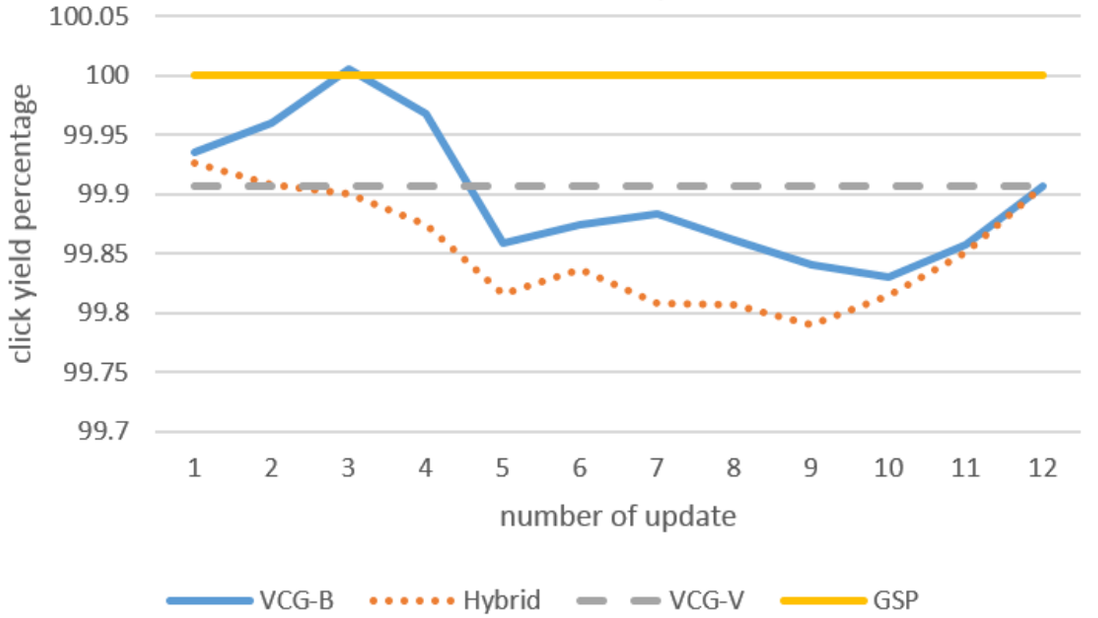
Auctions:

- GSP: GSP auction where bids are the bids in the dataset
- VCG-V: VCG auction where bids are the valuations
- Hybrid: auction proposed in our work where bids are given by mixed ads
- VCG-B: VCG auction where bids are given by mixed ads

Perfectly Rational Bidders







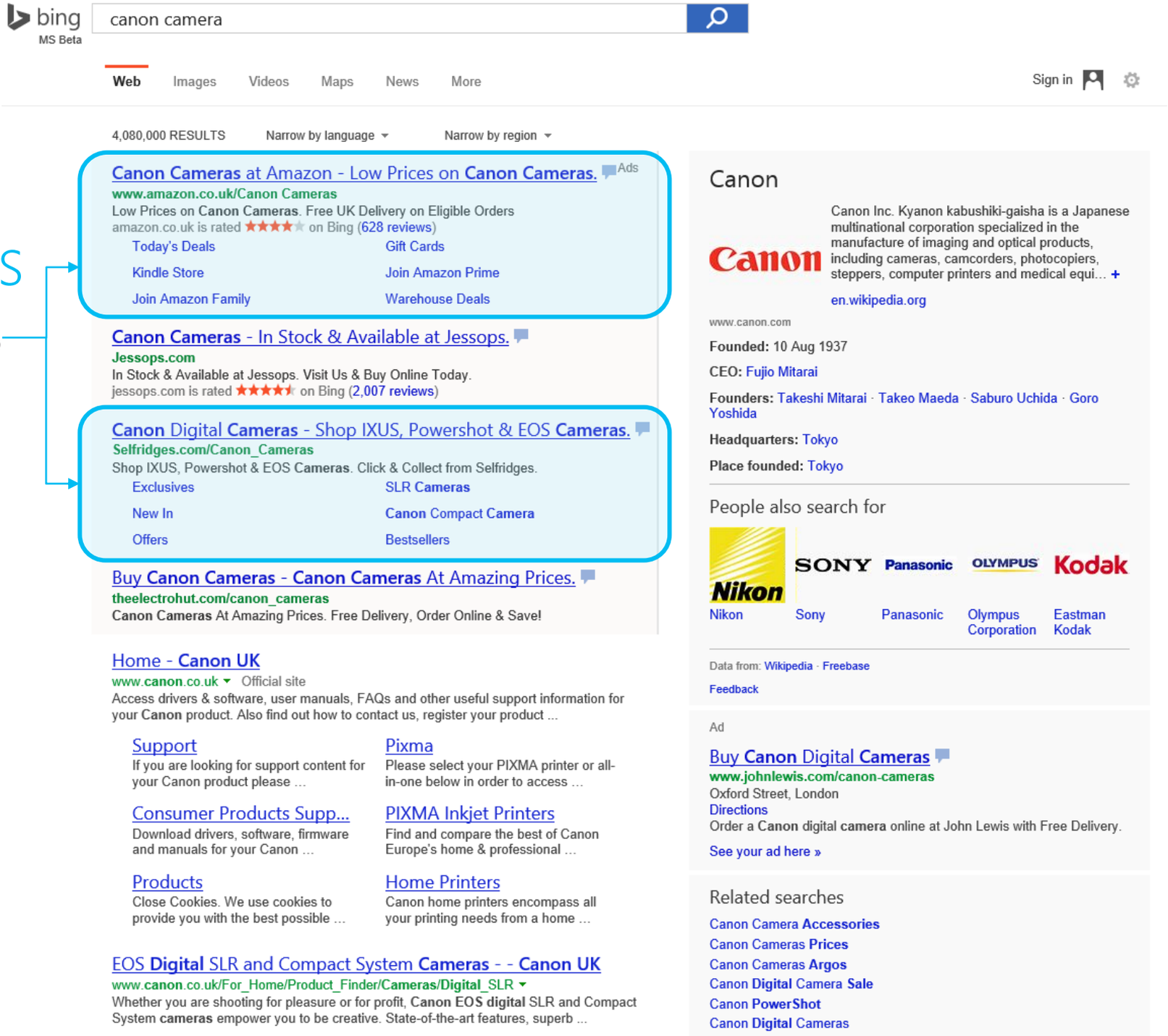
Thank You!

Ads Variants

ads with site links

images

…



Ads Variants

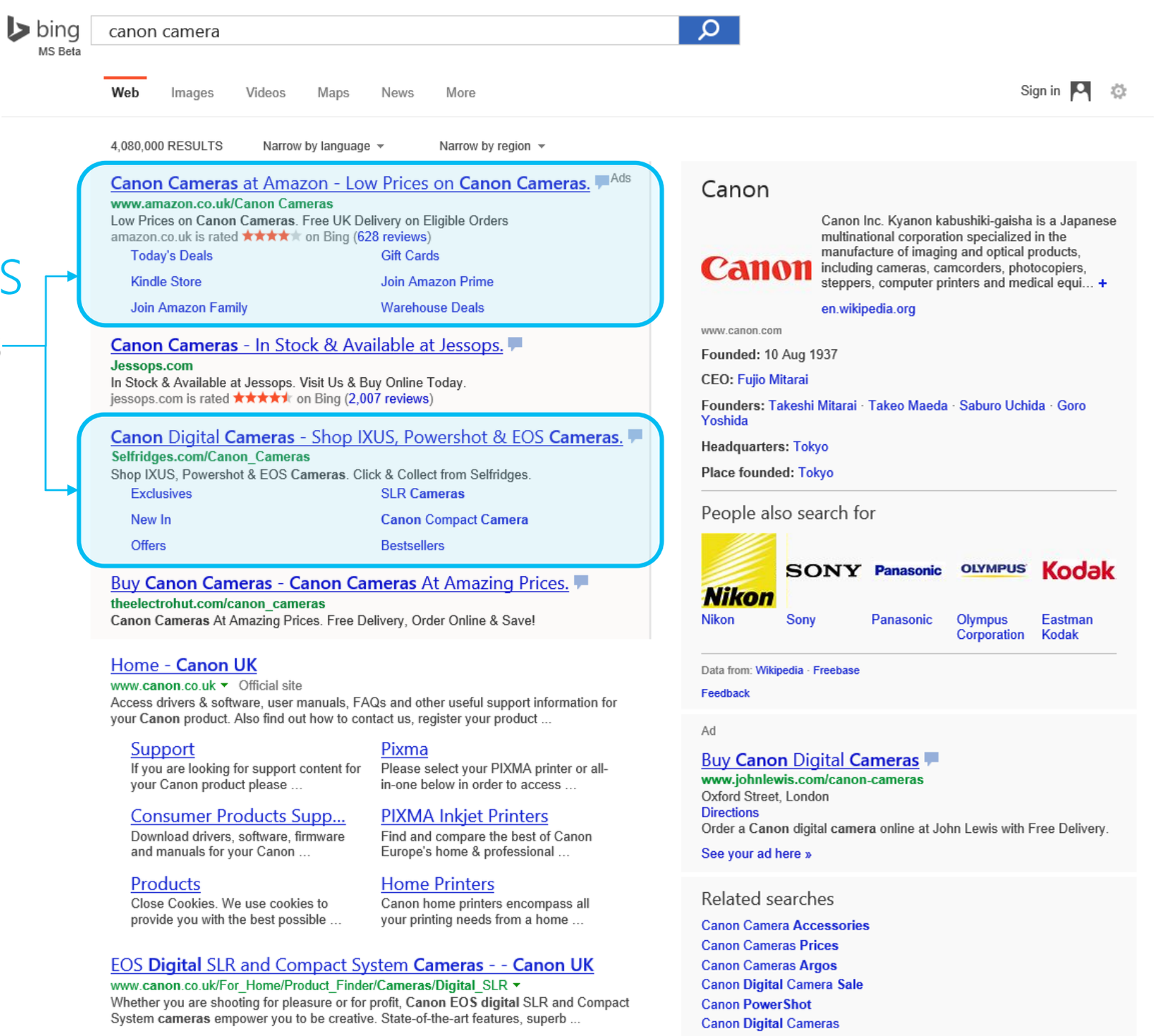
ads with site links

images

…

Templates

combinations of ads variants



GSP does not work!

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GSP is not well defined with Ads Variants and Templates

GSP generalizations fail

+ Depends on advertisers strategies
+ Hurts non-strategic (small) businesses

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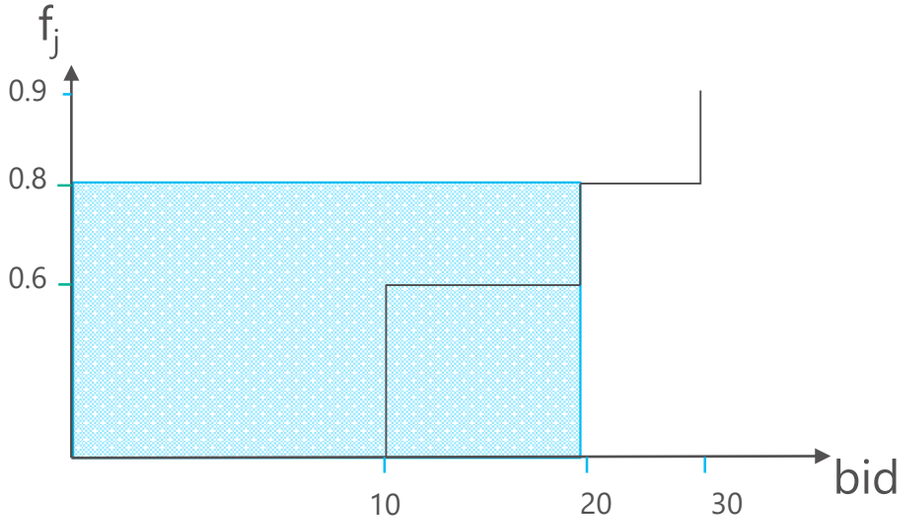

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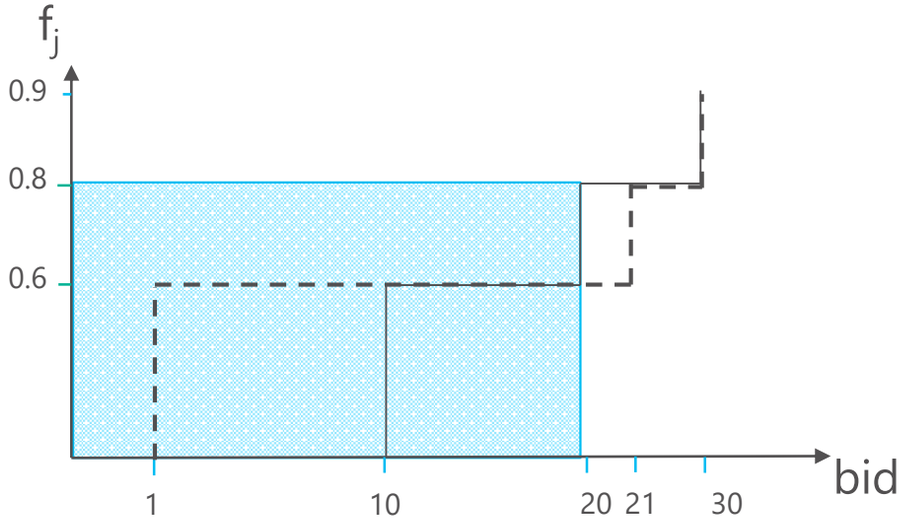

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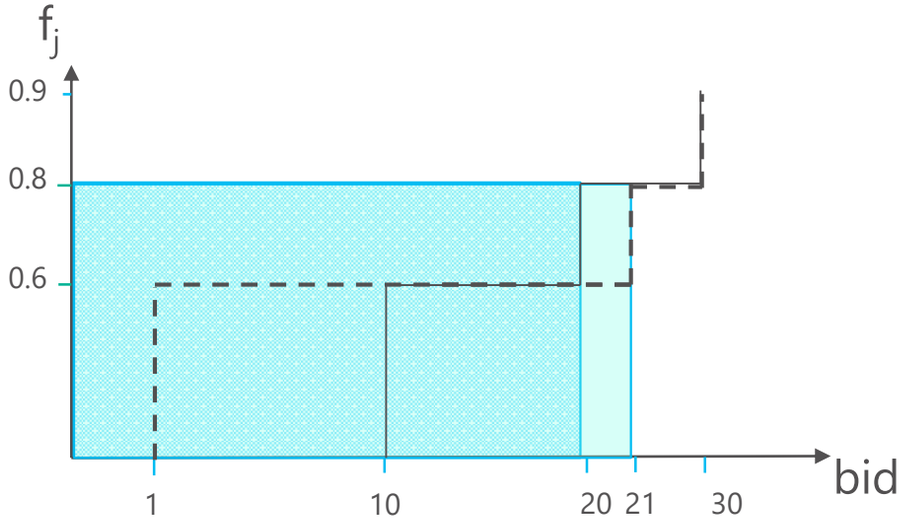

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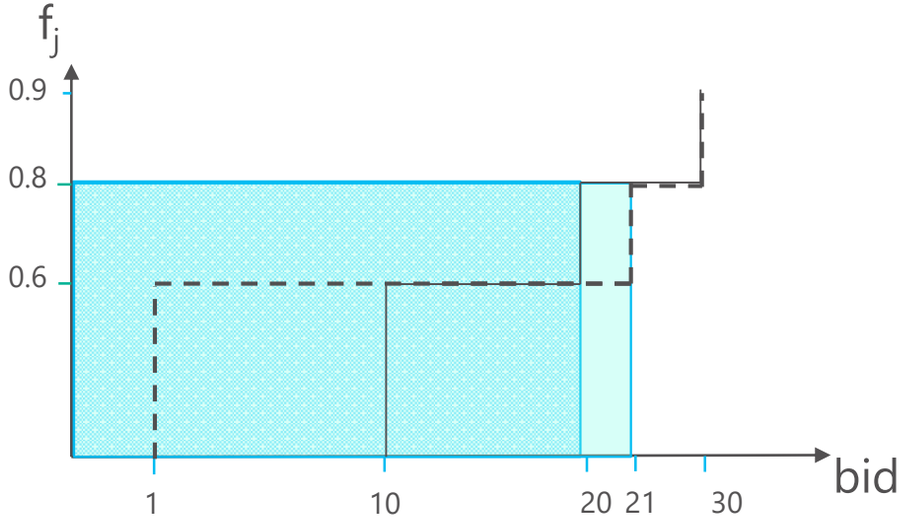

Which auction can be used?

Truthful auctions are the answer!

Can be used with Ads Variants and Templates

+ remove computational load from advertisers
+ make the market more transparent
+ same bid can be used across multiple auctions
+ easier analysis of the market

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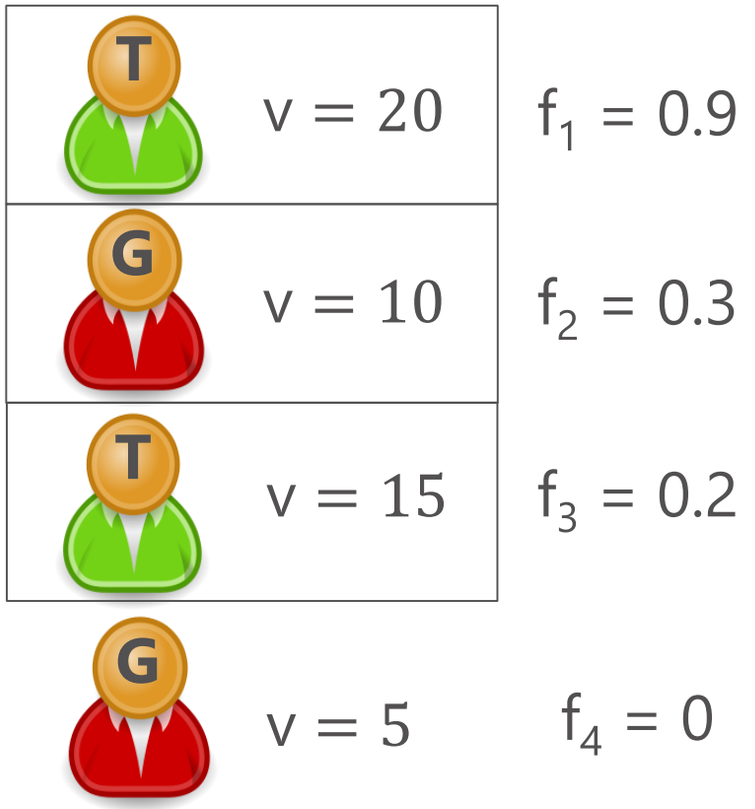
A truthful auction incentivizes advertisers to bid their value

Example of Payment Function

$$p^{(i-1)}(\Pi^{(i)}) = \max\Big(p^{(\arg\max v(\Pi^{(i)}))}(\Pi^{(\arg\max v(\Pi^{(i)})+1)})$$

$$+v_{max}(\Pi^{(i)})(f_{i-1} - f_{\arg\max v(\Pi^{(i)})}), b_{max}(\Pi^{(i)})f_{i-1}\Big)$$



Example of Payment Function

i-1 = 1

$$p^{(i-1)}(\Pi^{(i)}) = \max\Big(p^{(\arg\max v(\Pi^{(i)}))}(\Pi^{(\arg\max v(\Pi^{(i)})+1)})$$

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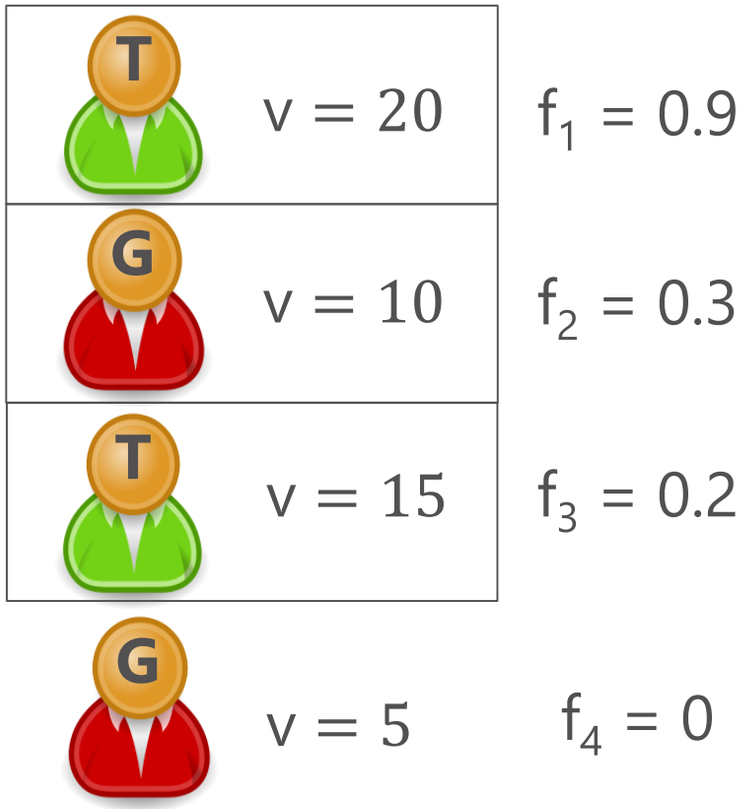
$p^{(1)} =$



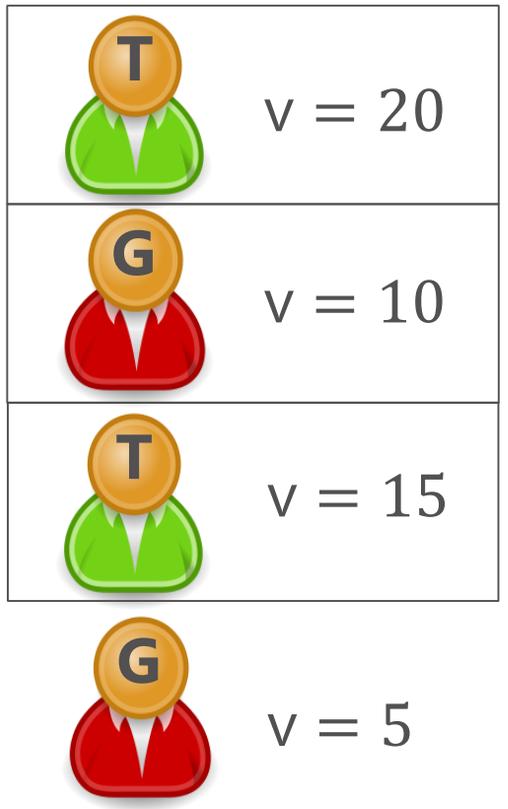
Example of Payment Function

slot below i with the T ad with the highest value: 3

payment of ad in slot 3

i-1 = 1

$$p^{(i-1)}(\Pi^{(i)}) = \max\Big(p^{(\arg\max v(\Pi^{(i)}))}(\Pi^{(\arg\max v(\Pi^{(i)})+1)}) + v_{max}(\Pi^{(i)})(f_{i-1} - f_{\arg\max v(\Pi^{(i)})}), b_{max}(\Pi^{(i)}) f_{i-1}\Big)$$



$p^{(1)}$ = max(5*0.2

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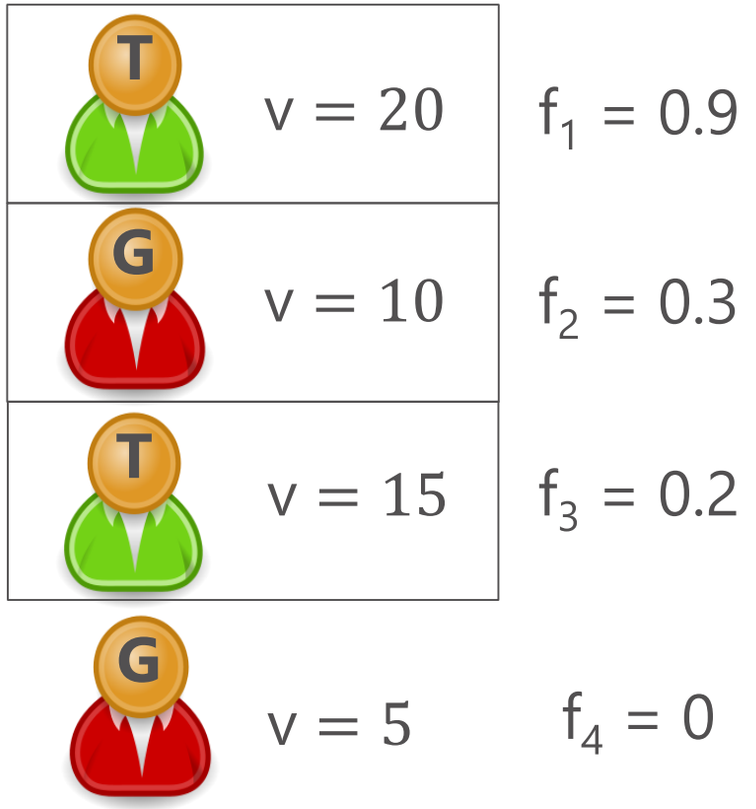
highest bid of T ad below position i-1: 15

$p^{(1)}$ = max(5*0.2 + 15 * (0.9 - 0.2),



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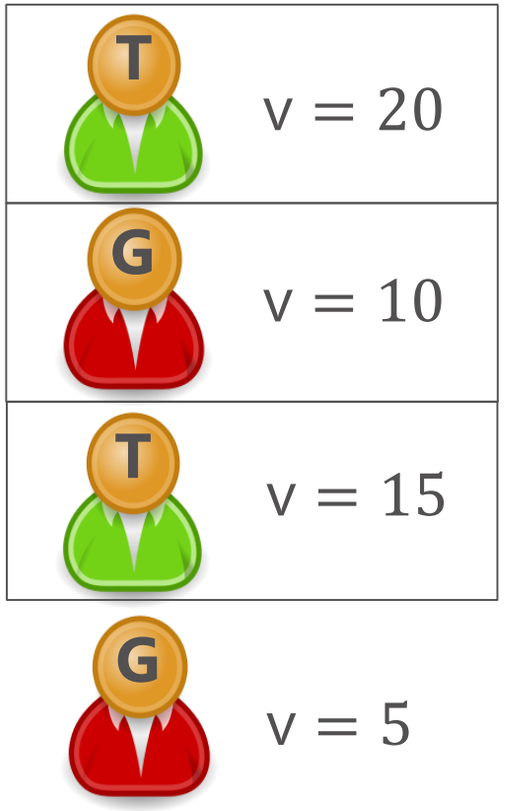
highest bid of G ad below position i-1: 10

highest bid of T ad below position i-1: 15



$p^{(1)} = \max(5*0.2 + 15 * (0.9 - 0.2), 10 * 0.9)$

$p^{(i-1)} = \max(p_j + v_j (f_{i-1} - f_j), b_k * f_{i-1}$