Ad-auctions: where Game Theory meets ML and CS

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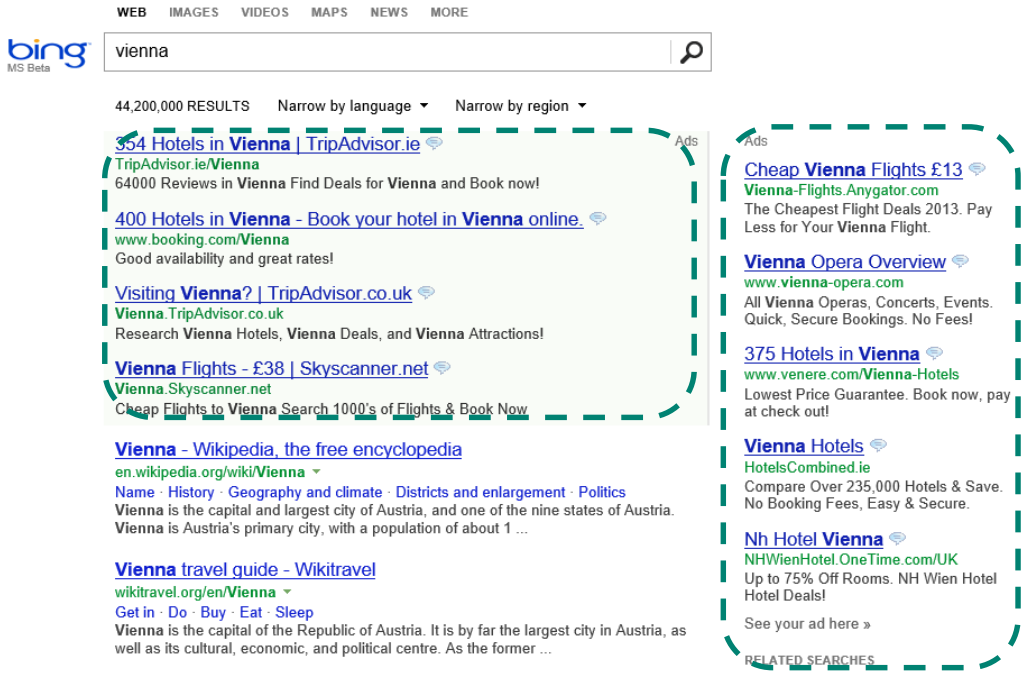
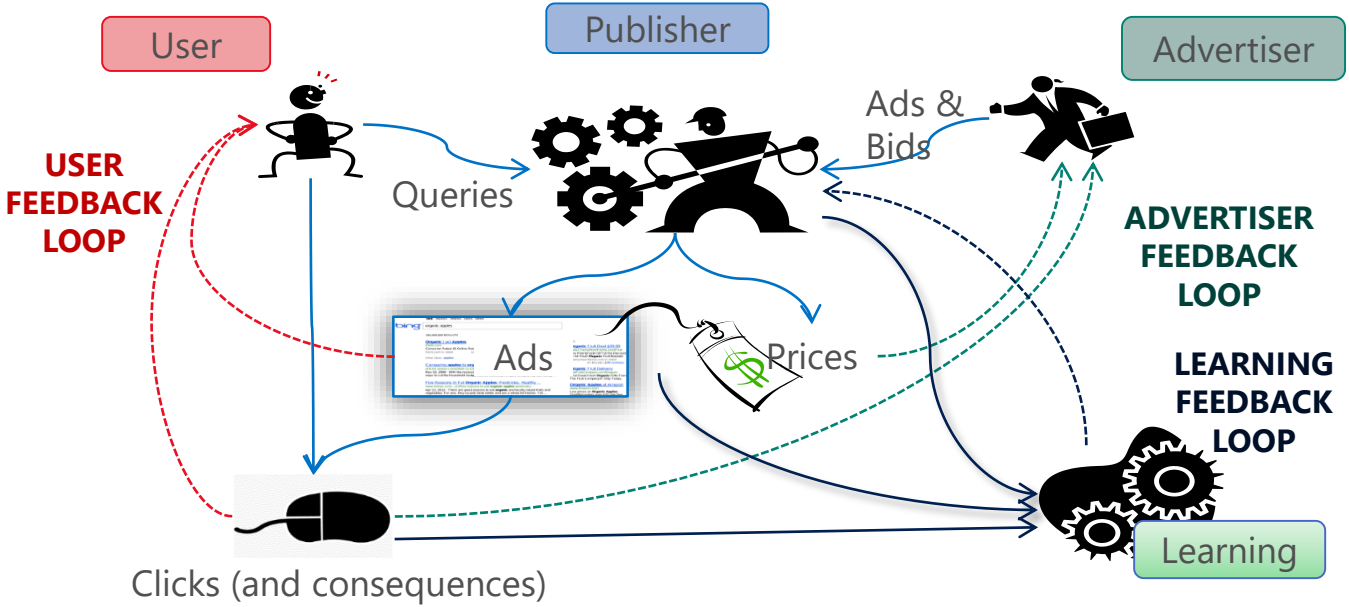
Some current electronic markets

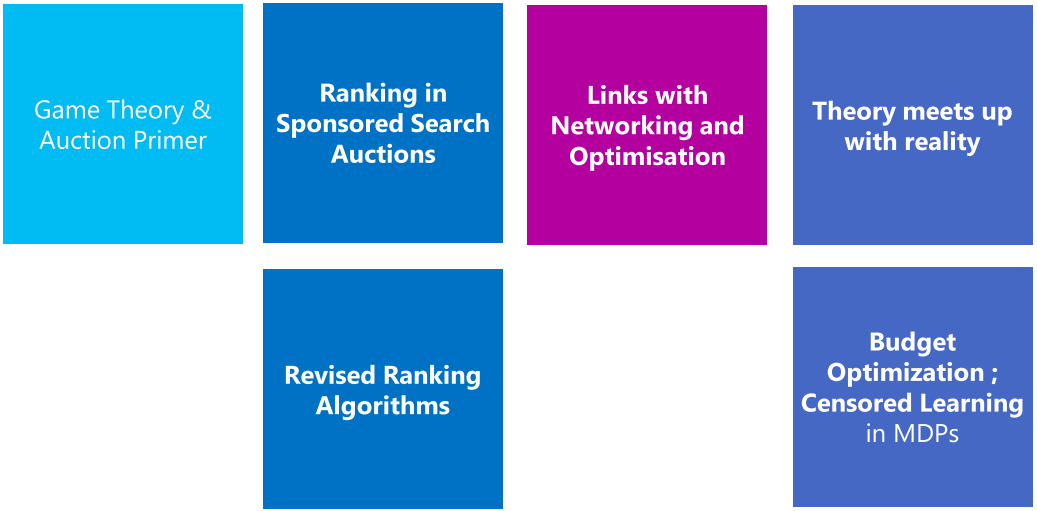
Those pesky little ads…



Three feedback loops, three stake-holders



RoadMap


Game Theory & Auction Primer
Ranking in Sponsored Search Auctions
Links with Networking and Optimisation
Theory meets up with reality
Revised Ranking Algorithms
Budget Optimization ; Censored Learning in MDPs

<auction theory>

Vickrey 2nd price auction
(VCG : Vickrey-Clarke-Groves)

Pricing

- One item: Pay "second price"
- Multiple items: Pay the "opportunity cost"

Properties

- **Efficient** – allocates to those who value items most
- **Incentive Compatible** — best to tell the truth

Drawbacks

- Doesn't maximise revenue (so use reserve prices, c.f. Myerson)

Myerson's Optimal Auctions

- Framework for truthful auctions, to maximise seller utility (revenue)
- Shows Revenue Equivalence:

  allocation fully defines prices
- Corresponds to reserve prices in simple iid private value setting

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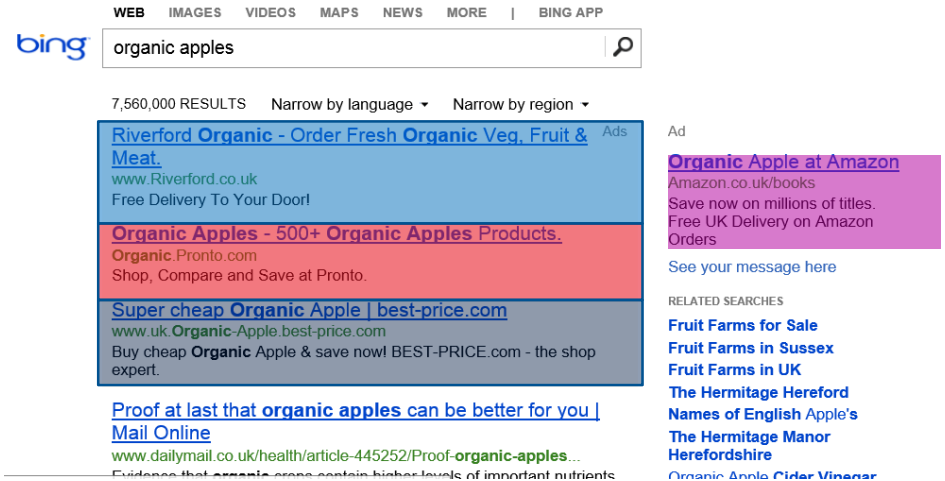
Ranking Ads


WEB IMAGES VIDEOS MAPS NEWS MORE | BING APP
bing
organic apples
7,560,000 RESULTS
Narrow by language
Narrow by region
Ads
Riverford Organic - Order Fresh Organic Veg, Fruit & Meat.
www.Riverford.co.uk
Free Delivery To Your Door!
Organic Apples - 500+ Organic Apples Products.
Organic.Pronto.com
Shop, Compare and Save at Pronto.
Super cheap Organic Apple | best-price.com
www.uk.Organic-Apple.best-price.com
Buy cheap Organic Apple & save now! BEST-PRICE.com - the shop expert.
Proof at last that organic apples can be better for you | Mail Online
www.dailymail.co.uk/health/article-445252/Proof-organic-apples...
Ad
Organic Apple at Amazon
Amazon.co.uk/books
Save now on millions of titles. Free UK Delivery on Amazon Orders
See your message here
RELATED SEARCHES
Fruit Farms for Sale
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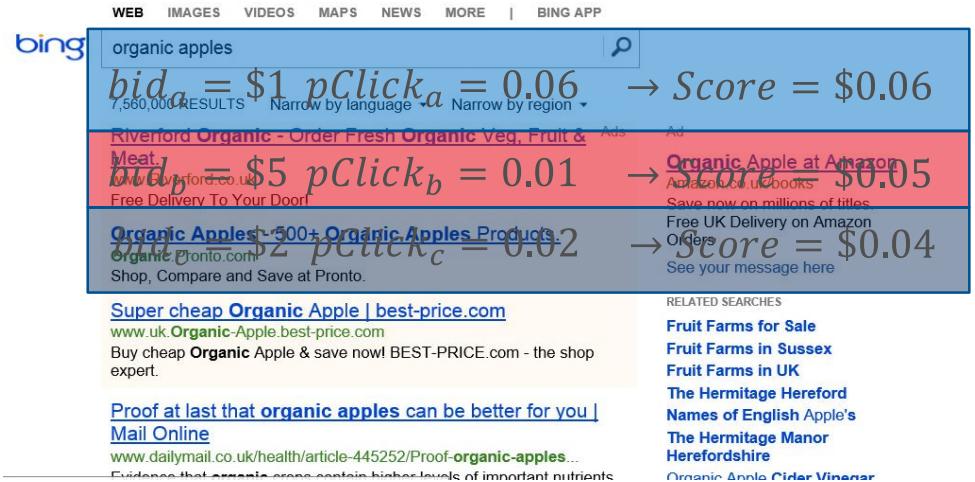
The Hermitage Hereford
Names of English Apple's
The Hermitage Manor Herefordshire

Ranking Ads

- GSP (Generalised Second Price) Ranking



From Ranking to Pricing

- GSP: Generalized Second Price Auction

"pay the bid you need to keep your slot"

$$Payment_i = \frac{bid_{i+i} \times pClick_{i+1}}{pClick_i}$$

- But… not truthful!

Standard "Theory" of GSP (Varian, EOS)

- Assume single one-shot auction
- Full information game: action=$\boldsymbol{bid_i}$, payoff=surplus
- Fixed slot effects, fixed number of bidders
- Theorem

A lattice of Symmetric Nash Equilibrium exists, and the lowest revenue SNE corresponds to the VCG mechanism

- *Def:* symmetric Nash or "locally envy free" equilibrium is where don't want to swap positions and payments with another

Revised Ranking - making trade-offs

Ben Roberts, Dinan Gunawardena, Ian Kash, and Peter Key, Ranking and TradeOffs In Sponsored Search, in *EC13: 14th ACM Conference on Electronic Commerce*, ACM, June 2013

Several Stakeholders

- Publisher: Revenue
- Advertisers: Clicks / Conversions
- Users: Relevance

Ranking Ads

- Original Yahoo! Method

Bid_a = \$5 $pClick_a$ = 0.01

Bid_b = \$2 $pClick_b$ = 0.02

Bid_c = \$1 $pClick_c$ = 0.06

- Original Google Method

Bid_c = \$1 $pClick_c$ = 0.06 → \$0.06

Bid_a = \$5 $pClick_a$ = 0.01 → \$0.05

Bid_b = \$2 $pClick_b$ = 0.02 → \$0.04

Making Tradeoffs

- Basic Ranking

Bid_c = \$1 $pClick_c$ = 0.06 → \$0.06

Bid_a = \$5 $pClick_a$ = 0.01 → \$0.05

Bid_b = \$2 $pClick_b$ = 0.02 → \$0.04

- Minimum Score of \$0.05

Bid_c = \$1 $pClick_c$ = 0.06 → \$0.06

Bid_a = \$5 $pClick_a$ = 0.01 → \$0.05

Making Tradeoffs

- Basic Ranking

Bid_c = \$1 $pClick_c$ = 0.06 → \$0.06

Bid_a = \$5 $pClick_a$ = 0.01 → \$0.05

Bid_b = \$2 $pClick_b$ = 0.02 → \$0.04

- Raise pClick to 0.75

Bid_a = \$5 $(pClick_a)^{0.75}$ = 0.03 → 0.15

Bid_c = \$1 $(pClick_c)^{0.75}$ = 0.12 → 0.12

Bid_b = \$2 $(pClick_b)^{0.75}$ = 0.05 → 0.10

How to Compare Rankings?

- Problem: multiplicity of equilibria
- Solution: generalise standard ranking [Varian / EOS result (where lowest SNE = VCG)]

Theorem: *For any GSP using a ranking of the form*

$$y(b, w) = \left(g(w)b - h(w)\right)^{+}$$

the set of SNE is non-empty and a lowest revenue SNE exists,

where $\boldsymbol{b}$ *is the bid,* $\boldsymbol{w}$ *is advertiser effect (so for ad* $\boldsymbol{i}$ *in slot* $\boldsymbol{j}$*,* $CTR = w_i s_j$ *)*

Optimal (Revenue) BNE Auctions

(Myerson, Maths of OR, 1981)

- User i value v_i , utility $U(v) = v_i x(v) - p(v)$
- Allocation $x(v)$ payments $p(v)$
- If Incentive Compatible $v_i = \arg\max_v U(v)$

implies U convex & $\frac{\partial U(v_i)}{\partial v_i} = x(v_i)$

$$\Rightarrow p(v_i) = p(o) + x(v_i)v_i - \int_0^{v_i} x(u)du$$

Optimal revenue (Myerson)

- If $v_i \sim F$ then

$$\mathbb{E}[p(v_i)] = p(0) + \int \left(v_i - \frac{1-F(v_i)}{f(v_i)}\right) x(v_i) dF(v_i)$$

$$= p(0) + \int \phi(v_i) x(v_i) dF(v_i)$$

where $\phi(v_i) = \left(v_i - \frac{1-F(v_i)}{f(v_i)}\right)$

is the "virtual valuation"

Myerson's Optimal Auction Theory

$$Revenue = \mathbb{E}\left[\sum_{i=1}^{n} \varphi(v_i) p_i x_i(\boldsymbol{v})\right]$$

Allocation function

where

$$\varphi(v_i) = v_i - \frac{1 - F(v_i)}{f(v_i)}$$

"Virtual Valuation"

Myerson's Optimal Auction Theory

$$Revenue = \mathbb{E}\left[\sum_{i=1}^{n} \varphi(v_i) pClick_i x_i(v)\right]$$

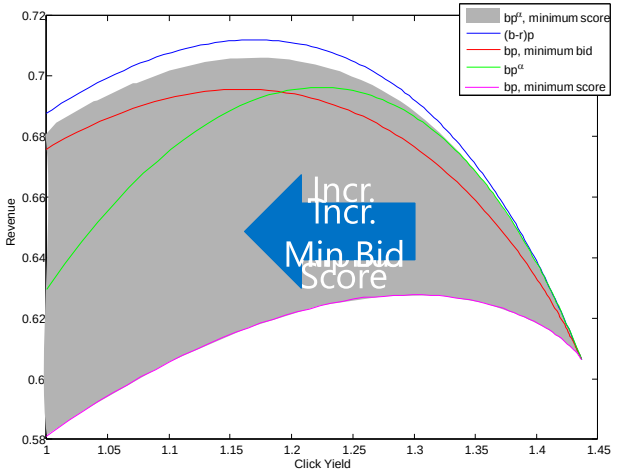
- Ranks ads using "Virtual Score" – $F(bid) \times pClick$
- Uses a minimum bid – $F(bid) < 0$ when $bid < r$
- Penalizes low bids – $F(bid)$ is small for bids near r

Ranking inspired by Myerson

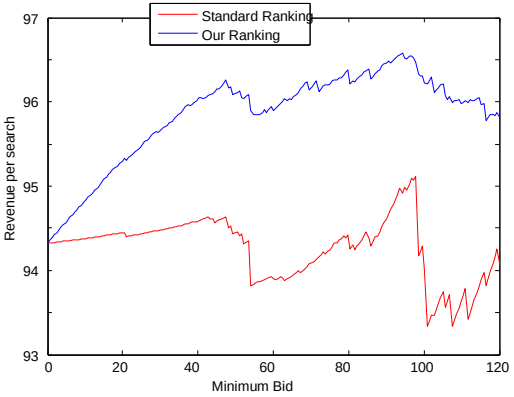
$$Revenue = \mathbb{E}\left[\sum_{i=1}^{n} \varphi(v_i) pClick_i x_i(v)\right]$$

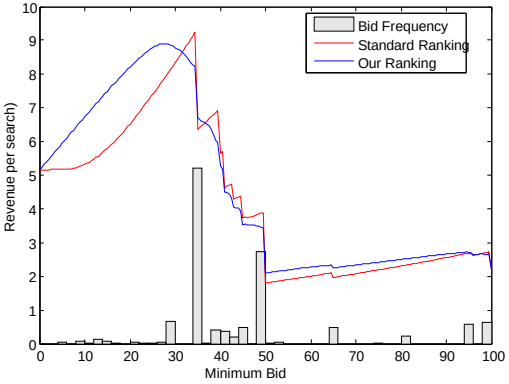
Our Proposed ranking: $F(bid) = bid - r$

Synthetic Data (Advertiser Reaction)



Actual Queries (No Advertiser Reaction)





Recap

1) [EOS, Varian, AGM] extend to many natural rankings

2) Optimal auctions do not rank ads efficiently

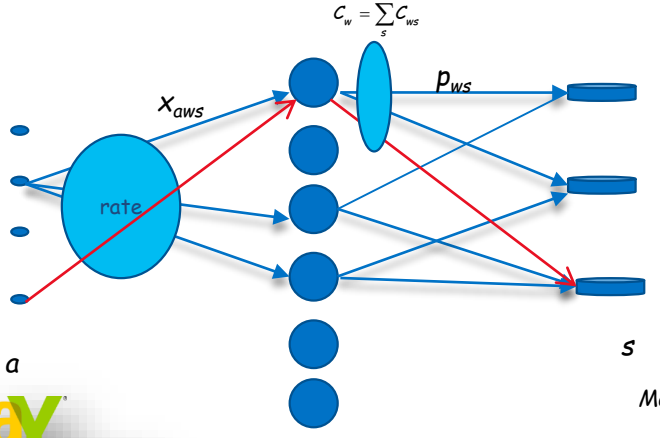
Links with networking and optimisation

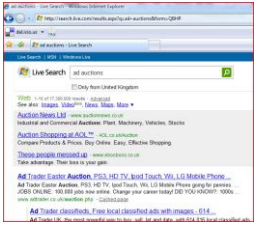
(with Neil Walton & Frank Kelly)

Auctions and Networks: commonalities

- How to allocate users/bid for slots/choose a channel or frequency
  - Non-stochastic MAB (minimise regret)
  - Exploration /exploitation trade off
- Users/agents/protocols adapt rates (bids) to maximise return in face of feedback signals

A MULTIPATH ROUTING EXAMPLE





$$Maximise \sum_a U_a(y_a)$$

$$s.t \quad \sum_{a,s} x_{aws} \le C_w$$

$$\sum_{w,s} p_{ws} x_{aws} = y_a$$

$$\sum p_{ws} \le 1$$

NUM differences

- Network:

$$\begin{aligned} \max \quad & \sum_i v_i \log x_i \\ st \quad & \sum_i x_i = 1 \end{aligned} \Rightarrow x_i = \frac{v_i}{\sum_i v_i}$$

- Position Auction

$$\begin{aligned} \max \quad & \sum_i v_i x_i \\ & x_i \le 1/L \\ st \quad & \sum_i x_i = 1 \end{aligned}$$

An different approach to Mechanism Design

- Start with the optimisation?
  - From optimization→ mechanism

- Advertiser $i, wtp\ \lambda_i$, search type τ, ad type l, ClickThrough Probablities p_{il}^{τ}

allocation $x_{il}^{\tau} \in [0,1], \sum_i x_{il}^{\tau} \leq 1\ , \sum_l x_{il}^{\tau} \leq 1\ ,$

An optimization approach

- Assignment $(\tau, l) \rightarrow (x^{\tau})$

  $\max \ \sum_l \lambda_i \, p_{il}^{\tau} x_{il}^{\tau}$ s.t. constraints met

- Advertiser $(U_i, y_i, i) \rightarrow \lambda_i$

  $$\min_{\lambda_i} U_i^*(\lambda_i) + \lambda_i y_i$$

  (U_i^* is Legendre-Frenchel transform of U_i)

- System $(U_i, I, \mathrm{T}) \rightarrow \{y_i, x_{il}^{\tau}\}$

  $\max \sum_i U_i(y_i) \ \ s.t. \ y_i \leq \mathbb{E}_{\tau}[\sum_l p_{il}^{\tau} x_{il}^{\tau}]$

Initial Results

- y^{τ}, x^{τ} optimal for SYSTEM if and only if there is a vector of prices λ^{τ} where assignments x^{τ} and click through rates y^{τ} are s.t λ^{τ} is optimal for the ADVERTISER for each i , x^{τ} is an optimal ASSIGNMENT for each τ, and $\lambda_i^{\tau} > 0 \Rightarrow y^{\tau} = Expected\ CTR$

Real World Optimisation Challenges

- Current ad-auction use on-line delivery with off-line optimisation
- Any *mechanism* needs to be tuned (e.g. reserve prices, constraints etc)
  - Problems are large --- currently clustering / aggregation needed
  - Current prevalent Map-Reduce framework not well suited to (mathematical) optimisation
  - Causal reasoning via counterfactuals

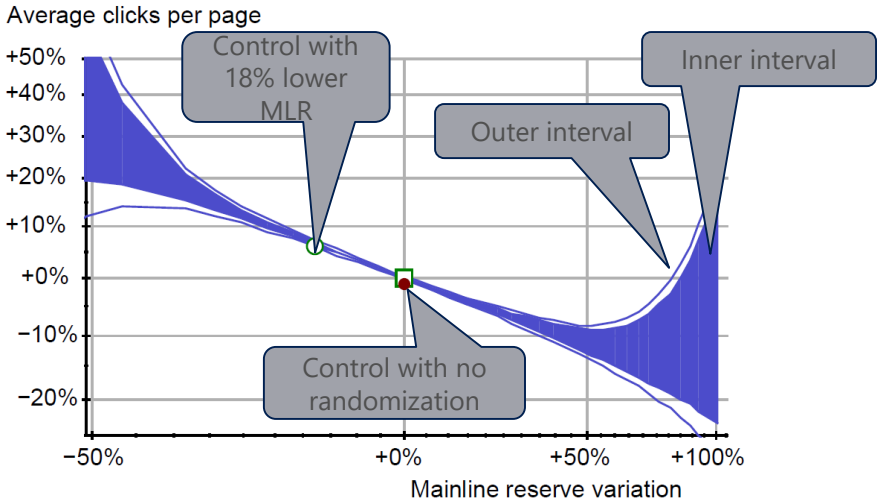
Counterfactual equilibrium

Counterfactual question

"What would have been the system performance metrics if we had applied an infinitesimal change to the parameter θ of the scoring model long enough to reach the equilibrium during the data collection period? "

Estimation technique similar to the *quasi-static analysis* in physics.

Playing with mainline reserves (ii)



Where models meet reality…

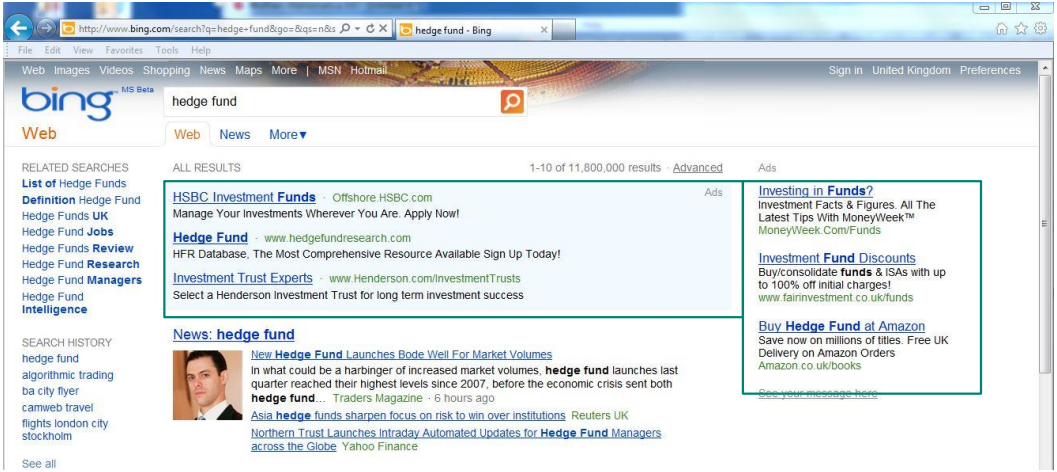
Traditional Approach: Game Theory

- Assumed click-through weights are known
- Number of bidders is known
- Bidders solve a Nash equilibrium. Unrealistic!

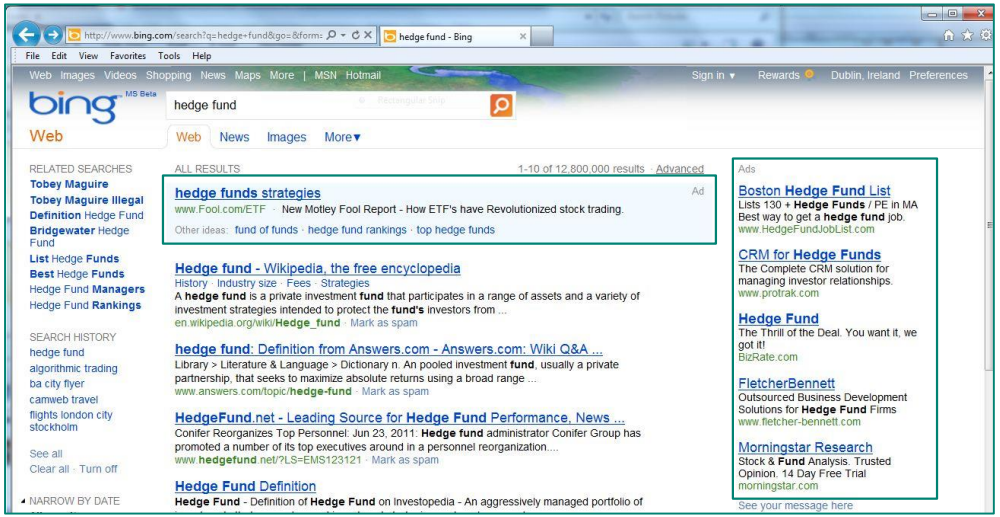
\	Bid 1	Bid 2	Bid 3	Bid 4
Bid 1				
Bid 2				
Bid 3				
Bid 4				

Sponsored Links

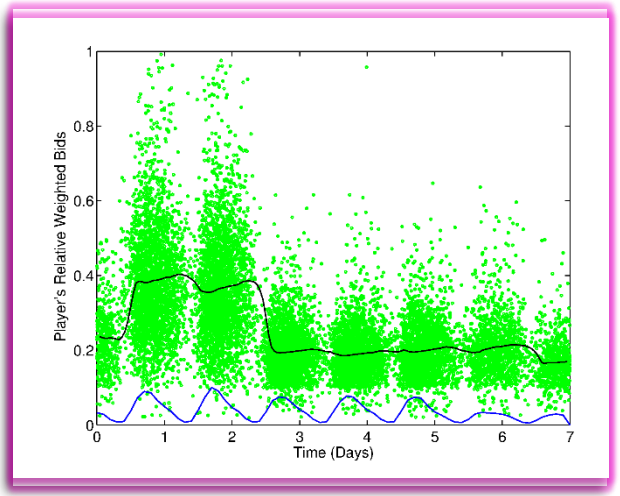
Location: UK



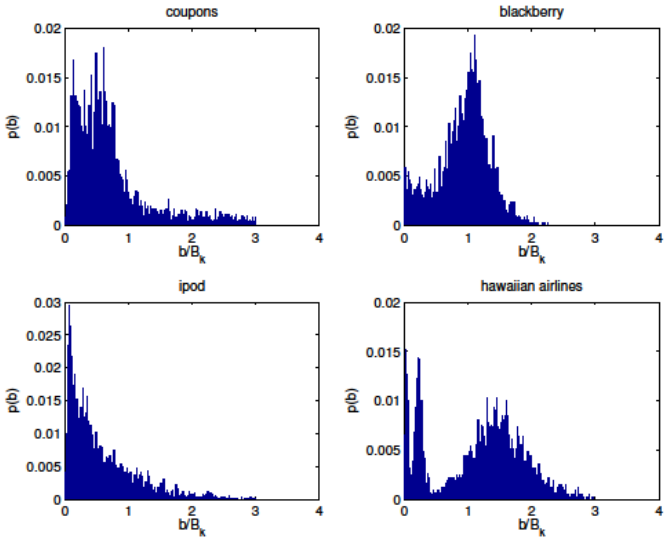
Location: Ireland



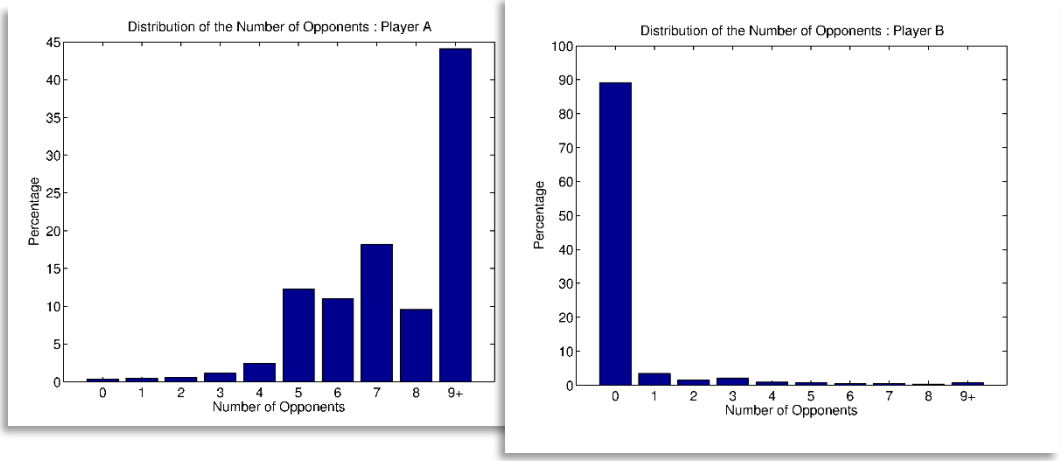
Click-through Rates Fluctuate A Great Deal



Bids and budgets

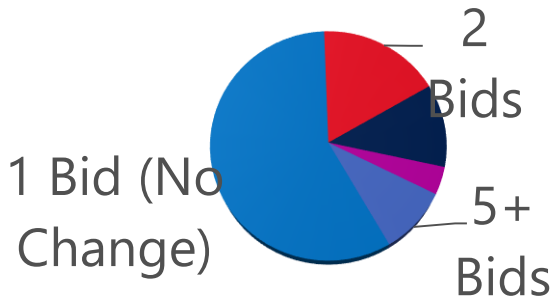


NUMBER OF OPPONENTS



Advertisers are rational?

- Most advertisers don't change their bid often



ANALYTIC MODEL

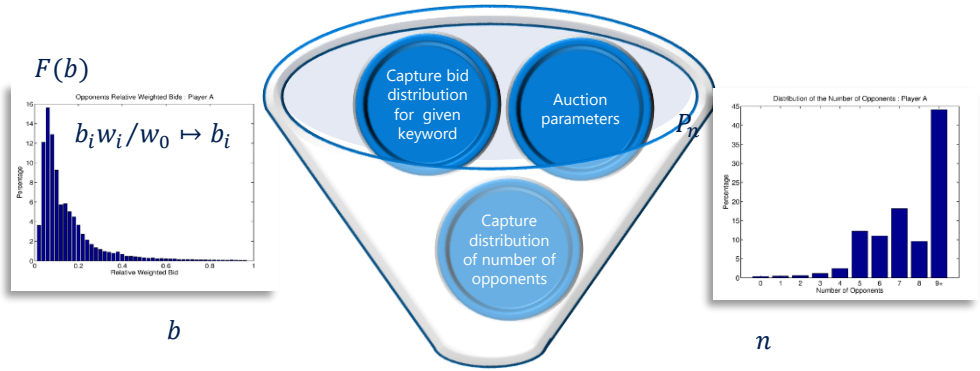
- Assumptions
  - Bidder "zero" face a random number of opponents n
    - $n \sim P_n$
  - Who place iid b bids drawn from distribution F
    - $b_k \sim F(\cdot)$
  - Allocation function $\varphi_i(b) \xrightarrow{slot} \{s_1, \dots, s_n\}$
- Probability n opponents and in position k :

$$\mathbb{P}(n \,\&\, \phi(b) = s_k) = p_n \binom{n}{k} F(b)^{n-k}\big(1 - F(b)\big)^k$$

- Expected Price when in position k :

$$\mathbb{E}(p(b)|n \;\&\; \varphi(b) = s_k) = b \; - \; \int_r^b \frac{F(t)^{n-k}}{F(b)^{n-k}} dt$$

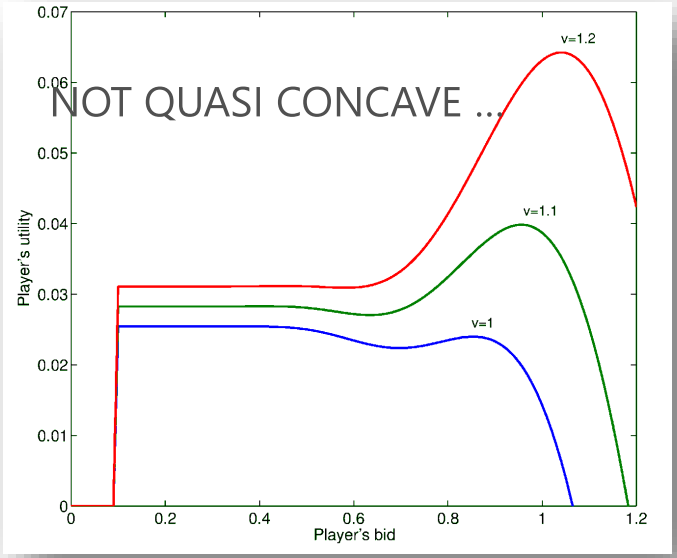
Value per click and optimal bid



Closed-form relation between optimal bid and value

$$v = b + \frac{\sum_{n=1}^{N}\sum_{k=0}^{n-1} p_n \binom{n-1}{k} s_{k+1} \left(1-F(b)\right)^k \int_r^b F(t)^{n-k-1} dt}{\sum_{n=1}^{N}\sum_{k=0}^{n-1} p_n \binom{n-1}{k} (s_k - s_{k+1}) \left(1-F(b)\right)^k F(b)^{n-k-1}}$$

UTILITY


NOT QUASI CONCAVE …
v=1.2
v=1.1
v=1
Player's utility
Player's bid
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
1.2

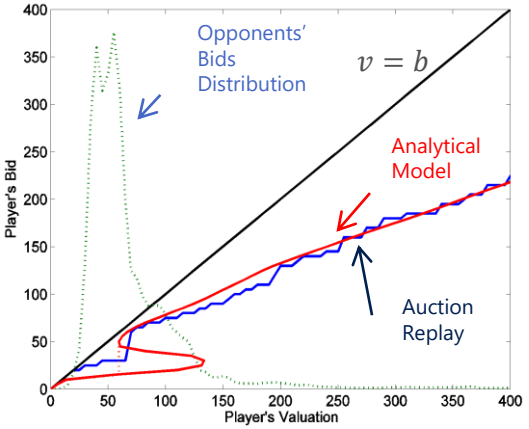
Results and validation

Optimal Bids and valuations calculated (F#)

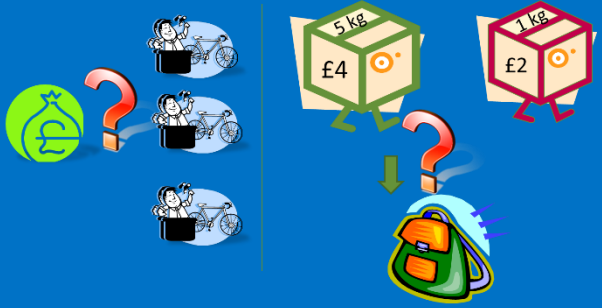
Validated against real clicks & estimated CTR

Single 8 core computer estimated 15777 ad valuations in 10 minutes

Practical for advertiser feedback



Bidding with Budgets



Optimal Bidding Strategies in Dynamic Auctions with Budget Constraints (on SSRN)
Ramki Gummadi (Stanford) Peter Key (Microsoft Research) , Alex Proutiere (KTH)

How should budget constrained advertises bid?

- By applying simple ("wrong") model that advertiser face opponents *as if* sampled from an iid distribution

1. Can find optimal bidding strategy in limiting regime
2. Can use censored learning to estimate distribution and use in online learning

Bidding with budgets

- Budgets constrained bidders
- Bid to maximise return in repeated auction
- Equivalent to online stochastic knapsack

Bidding with budgets

- Single good, repeated auction
- User has budget B_i(t), value $v_i(t)$, bids $a_i(t)$
- Winner pays 2nd highest price $p(t)$
  - Private valuation $\sim v$ (observable)
  - Competing bid $\sim p$ (not observable)
- Decision variable is bid at time t: $a(t)$
  - Depends on both $v(t)$ and $B(t)$
- Gain per unit time: $g(t) = \big(v_i(t) - p(t)\big)\mathbf{1}_{a_i(t)>p(t)}$

Bidding with budgets

- Objective: maximise discounted return

$$\sum_{t=0}^{\infty} e^{-\beta t}\mathbb{E}[g(t)]$$

- Assume opponents bids $p \sim iid\ F$

- Large system scaling: Scale both "time" and "space" as $\beta \to 0$

$$V_\beta(B) \triangleq \beta v_\beta(b) = \beta v_\beta\left(B/\beta\right)$$

Main insights

- *Shade* bid:

  shade(remaining balance B) $\triangleq \frac{1}{1+V'(B)}$

  so bid $\frac{v}{1+V'(B)}$

- $V(B)$ can be computed explicitly in the limit

- Extends other auctions SP, GSP, FP, etc. (any quasi linear utility)

Mean Field Games

- Large number *N* of competing bidders
- Divide into *K* classes, budget B_k, after $N \times T_k$ timeslots, budget re-initialised.
- Time *t*, select random subset $S_t \subset N$, size *S*

Definition: Restrict attentions to strategies that are the same modulo restarts. A *renewal* equilibrium, is an equilibrium in this setting.

Mean field equilibria

- As $N \to \infty$, distribution of opponents bid stationary
- Let *z* be dist. of highest $S-1$ bids
- $x_i(z) \triangleq$ optimal strategy of *i*,
- $\bar{\hat{x}}_i(z) \triangleq$ empirical bid distribution of *i*,

A MFE exists if $z \sim Max[\bar{\hat{x}}_i\,(z), \mathrm{i} = \pi(1), \ldots \pi(\mathrm{S}-1)]$

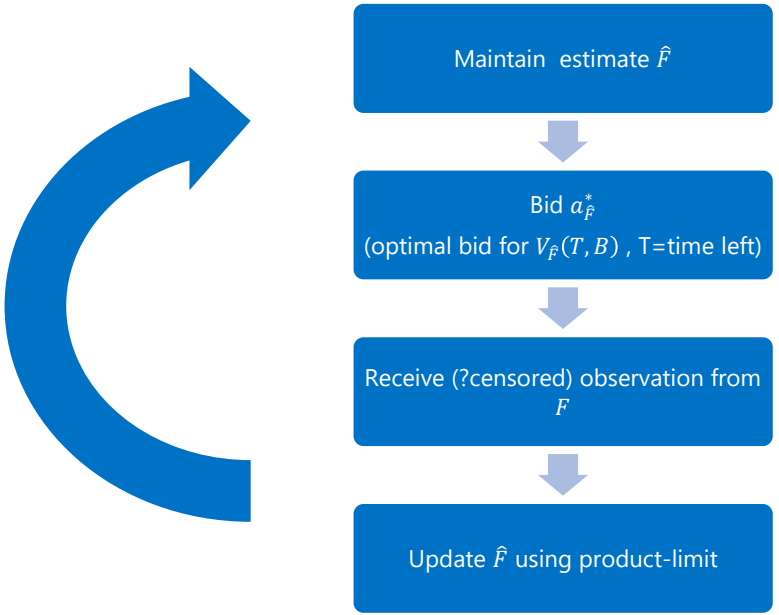
Theorem:

A MFE exists when there a lot of auctions per period.

Censored learning

- But opponents bid distribution unknown?
- Censored observations
  1. Imperfect observation of market price
     - If win with bid *b*, know price *p*
     - If lose, know price $p > b$
  2. Only charged if add clicked
  - Use Kaplan-Meier (product limit)

Greedy-Product limit Algorithm



Experiments

1. Distributional.
   - ❖ Construct distribution from empirical AdCenter distribution of bids.
   - ❖ Simulate model.
2. Sequential.
   - ❖ Violate Modelling Assumption.
   - ❖ Feed historical bids to algorithm sequentially.

Sequential: Compared to offline-optimal

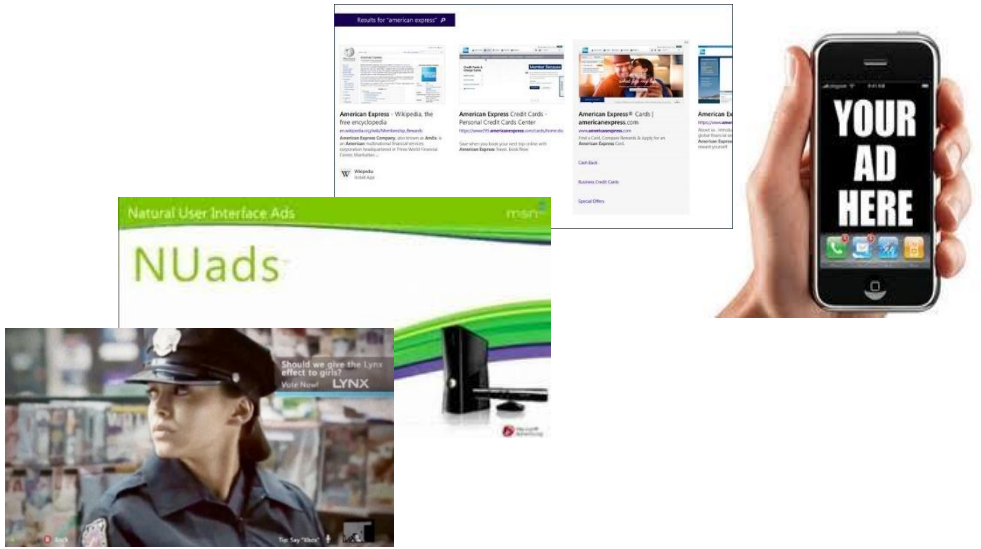
Name	Competitive Ratio	St dev
Greed PL	**0.91**	**.117**
LuekerLearn	0.90	.115
Fixed-Price search	0.63	.140
Q-Learn	**0.59**	**.156**
Budget Smooth	0.31	.325

Challenges and Research

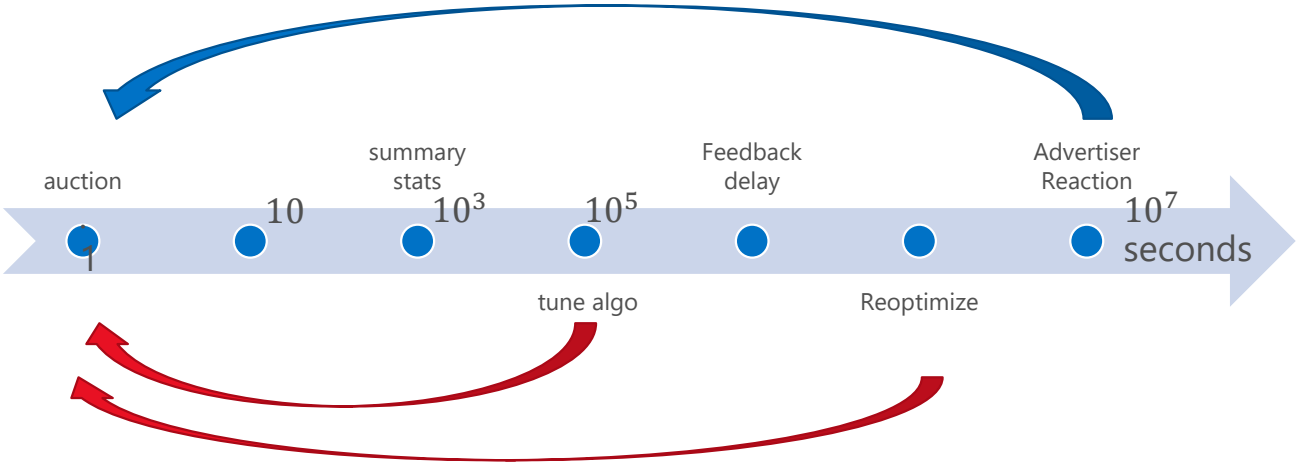
Auction Theory : state of the art

- Only simplest auctions are “well understood”
  - Single good, multiple identical goods
- In theory can design “truthful” auctions
  - Complex to implement, has other “bad” properties
- Optimal auctions: gives some insights
- Need a workable theory to deal with complexity on online services (repeated auctions) and trade-offs

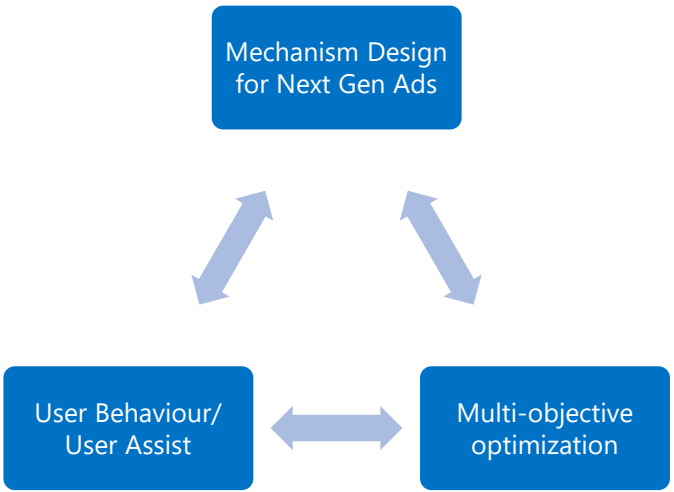
Ads on new platforms & Richer Ads



Optimization Timescales



Research areas

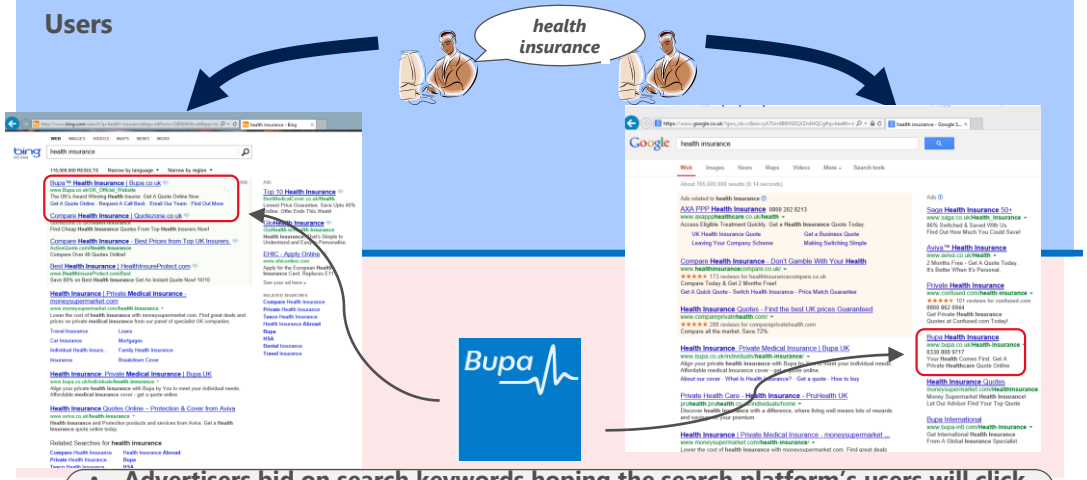


- Mechanism Design
  - For rich ads / extensions
- Multi-objective optimization
  - For different stake-holders
  - For page/"in app" optimization
- User Behaviour
  (Searchers and Advertisers)
  - To predict, for design
  - To influence/assist

Appendix

At the heart of search is a multi-sided market

- Users conduct search queries at no financial cost
- Platforms compete for these users in quality and innovation



- Advertisers bid on search keywords hoping the search platform's users will click their ads
- Advertisers allocate budgets between two search engines
- Platforms compete for advertisers' engagement and spend

User Behaviour

- User (searcher) behaviour for richer ads (Rich Ads / Ad Extensions) not well understood
  - Is top-to-bottom, left-to-right decreasing value gradient still true?
  - Understanding incentives necessary for mechanism design
- Advertiser reaction to (changing) mechanism design poorly understood
- Requires experimental/behavioural economics, Multi-Agent System theory & Game Theory

User assistance: Machine Learning & Game Theory

- Data can be mined for
  - Intelligent bid agents
  - Campaign management
  - User targeting
- Auto-generated ads
- Recommender systems
- Who “owns” the user data?

Multi-objective Optimization

- Multi-objective optimization needed:
  - Platform / Mechanism design needs to account for different stakeholders:
    - Publisher, advertisers, users, third parties & constraints (budgets etc.)
- Move from offline-online(delivery) to more online
- New system architectures needed:
  - Current MapReduce/ Hadoop ill-suited to optimization tasks
  - Distributed local optimization?
- Requires optimization/algorithmic/systems expertise